Multiuser Resource Allocation Optimization Using Bandwidth-Power Product in Cognitive Radio Networks

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***Abstract*—In this paper, the problem of resource allocation optimization is studied for a single-cell multiuser cognitive radio network in the presence of primary user networks. The spectral access of the cognitive radio network is based on Orthogonal Frequency Division Multiple Access (OFDMA). A joint bandwidth and power allocation is performed so that users' rate requirements are satisfied, and the integrity of primary user communication is preserved. In this work, two unique challenges are addressed. The first is the incorporation of primary user activity in the design of resource allocation technique, and the second is the limited hardware capabilities of cognitive terminals compared to those available at the cognitive base station. To address these problems, a novel resource allocation framework is proposed based on the bandwidth-power product minimization, which is an effective metric in evaluating the spectral resource consumption in a cognitive radio environment. The framework takes into consideration the challenges aforementioned. The results show significant enhancement in spectral efficiency by using our framework compared to classical power adaptive optimization using iterative waterfilling scheme.**

***Index Terms*—Cognitive Radio, Resource Allocation, Downlink, Optimization, Bandwidth-Power Product, OFDMA**

I. Introduction

SPECTRUM resource allocation for cognitive radio networks (CRNs) presents many unique challenges. One of these challenges is the mutual interference between Primary Users (PUs) and Cognitive Radio (CR) users [1], [5], [7], [22]. In this context, resource allocation must maximize the efficiency of the spectral resources utilization and minimize the risk of overlapping the coverage of CRNs with adjacent primary networks. Based on the seminal work of [8], the wireless network resources are characterized in bit-meters per second, which indicates that a joint optimization of bandwidth and power allocation is necessary for efficient resource utilization. It was found that the heterogeneity of the spectral footprint of cells in cellular networks is the key behind maximizing the coverage in a given area. The spectral footprint in this context is measured by the space-bandwidth metric suggested by [13]. Therefore, this metric can be an effective utility function for CRN resource allocation. Another challenge is imposed by the limited capabilities of wireless terminals associated with the cognitive base stations (CR-BS) in a centralized CRN architecture. The range of accessible bands by the BS is wider than the accessible range for its associated wireless terminals. This is because of the limited processing capabilities and power budget for wireless terminals. Current commercial wireless terminals can support one, two or three bands at most. GSM networks, for example, operate in two bands; namely 900MHz and 1800MHz bands. Many HSPA mobile phones can access bands I/II/V (2100,1900 and 850 MHz). As a result, it is desirable to investigate the performance of resource allocation in CR-BS with a heterogeneous set of wireless terminals in terms of its supported spectral bands.

Many studies on resource allocation for OFDMA wireless systems have been reported in the literature. The resource allocation schemes can be classified into two categories: margin-adaptive (MA) and rate-adaptive (RA) [19]. A survey on the resource allocation algorithms for OFDMA systems can be found in [20]. However, those resource allocation algorithms do not consider the unique nature of CR environment. Nevertheless, several studies have evaluated their performance in CRNs by adding the PU interference limits as an additional transmission power constraint [2], [16], [21]. In [14], a step further in the level of interaction between PUs and CRNs by developing a spectral sharing approach based on the primary radio network "willingness" metric, in which the PUs provide a small assistance to enable CRN dynamic spectrum access and real-time CR user-to-PU interference control.

In addition to the throughput and total power metrics used in the resource allocation optimization, the space-bandwidth product (SBP) have been proposed as an effective metric for CRNs. For example, SBP was used as the optimization objective function to regulate the spectrum sharing in multihop CRNs [11]. Yet, the transmit power spectral density was fixed in this study and the spectral footprint was controlled through the consumed spectral bandwidth. This metric can

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be further exploited in centralized CRNs. To the best of our knowledge, no studies have been done to examine minimizing the spectral footprint of such networks as an optimization objective of radio resource allocation for these networks.

Different from all the aforementioned work that considers classical network performance metric such as total transmission power or sum rate as the objective of the resource optimization process, our work is fundamentally based on a new metric that is shown to be more suitable for CRN environment. The contributions of this work include:

- Proposing the *bandwidth-power product* as an effective utility function for OFDMA based resource allocation optimization in CRN settings.
- Incorporating the *awareness* of PU activities in the proposed framework of resource allocation for CRNs.
- Incorporating the impact of limited number of *accessible bands* by CRN terminals into the resource allocation optimization framework.
- Exploiting *beamforming* in the formulated framework in order to further improve the frequency reuse in dense cognitive cellular networks.

The remainder of this paper is organized as follows. Section II develops the mathematical model and resource constraints for the wireless environment of the CRN. Section III formulates the resource allocation optimization as a mixed integer nonlinear programming problem. Then, Section IV proposes an iterative solution based on decomposition theory. Section V presents simulation results and demonstrates the performance of the proposed solution in comparison to other optimization solutions proposed in literature. Section VI concludes this work.

II. System Model

We consider the downlink resource allocation for a CRN based on OFDMA technology. Similar analysis can be conducted for the uplink resource allocation problem. The CRN is comprised of a CR-BS and a set J of CR users associated with it in the presence of one or more primary networks. Let us denote $\mathcal{P}$ as the set of coexisting primary networks. The CR-BS supports a frequency reuse factor of C. In other words, the CR-BS is capable of supporting up to C sectors within its coverage area. Hence, we define J_i as the set of users that belongs to sector $i \in \mathcal{C} = \{1, 2, ..., C\}$, $\cup J_i = J$, $\cap J_i = \phi$. Fig. 1 shows an example of the communication system in consideration. Suppose that the total number of channels available for allocation is K and the bandwidth of each channel is B. These channel resources are shared with primary networks. Therefore, any vacant channel that is borrowed by the CRN is subject to the risk of loosing its full capacity upon PU's return.

In such highly dynamic environment, the objective of the resource allocation in CRN is to minimize the utilization of wireless resources while maintaining a given QoS requirements of CR users (expressed in terms of required data rates) and CR- BS transmission power constraints. The resource allocation should adapt to the PU activity in two folds: (i) Frequency: by avoiding frequently-used frequency channels, and (ii) Power: by minimizing the interference caused by

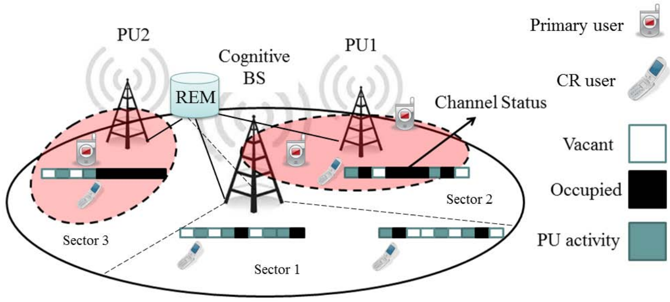


Fig. 1. Cognitive radio network deployment scenario. Radio Environment Maps (REM) is used for maintaining the records of PU activities

unreliable information about the channel condition. In this study, we measure the spectral resource utilization by the *product of bandwidth and transmission power*.

In the following subsections, we describe the wireless channel model and the set of interference events considered in the resource optimization framework.

A. Wireless Environment Model

The wireless signal propagation model is based on the outdoor COST-Hata propagation loss model [6]. Let $g_{i-j}^{(k)}$ be the channel gain between a BS i and its associated user j on channel k . We assume slow fading channel such that the channel gain is stable within the resource optimization interval. This assumption is justified in Section II-C. For simplicity, we omit the transmitter subscript in the channel gain if the transmitter is the CR-BS. Hence, we denote the channel gain between the CR-BS and user j as g_j, the channel gain between the CR-BS and a PU node p as g_{PU_p}, and the channel gain between PU node p and CR receiver j as g_{PU_p-j}. The value of g_j is updated through frequent measurements from the CR user. g_{PU_p} can be estimated under the channel symmetry assumption by evaluating PU signal at the CR-BS. g_{PU_p-j} is not directly estimated. Instead, the collective interference caused by the adjacent primary network is estimated as discussed in the next section.

B. Primary User Interference Characterization

As a result of the coexistence between the CR-BS and other primary networks, we define two sets of interference:

- CR-PU Interference: This is the CR-BS interference on a primary network. Assuming that the CR-BS signal is an OFDM signal, the interference caused by this BS on the primary network can be modeled as follows [22]:

$$\mathcal{I}_p^{(k)} = |g_{PU_p}^{(k')}|^2 \int_{d-\frac{B}{2}}^{d+\frac{B}{2}} p_j^{(k)} T_s \Big(\frac{\sin(\pi f T_s)}{\pi f T_s}\Big)^2 \quad (1)$$

  where d is the spectral distance between channel k used by CR and channel k' used by PU. This interference expression takes into consideration adjacent channel interference effect. In case of a complete overlap between PU and CR channels, channel k' is the same as channel k and correspondingly d is equal to zero. T_s is the OFDM symbol period, $p_j^{(k)}$ is the transmission power of user j on channel k.

- PU-CR interference: This is the primary network interference on the CR user j. Since the waveform of the PU activity is unknown, we assume that $\rho_{PU}^{(k')}(f)$ is the power spectral density of PU signal at channel k', then the interference of that signal on the CR that is using channel k user can be calculated as follows:

$$\mathcal{J}_{pj}^{(k)} = |g_{PU_p-j}^{(k)}|^2 I_{pj}^{(k')} \tag{2}$$

where $I_{pj}^{(k')} = \int_{d-\frac{B}{2}}^{d+\frac{B}{2}} \rho_{PU}^{(k')}(f)df$.

Based on the signal propagation path loss and interference models above, we can formulate the channel gain to interference and noise ratio (CINR) at CR user j at channel k as follows:

$$h_j^{(k)} = \frac{g_j^{(k)}}{\Gamma(\eta + \sum_{p\in\mathcal{P}} \mathcal{J}_{pj}^{(k)})} \tag{3}$$

where η is the ambient Gaussian noise and Γ is the signal-to-noise ratio gap to Shannon capacity limit [10], [23].

C. General characteristics and assumptions

We list and discuss the assumptions used in developing the resource allocation framework:

A1: Sensing: The CINR values are obtained by spectrum sensing operation. The sensing operation can be done via a dedicated sensing receiver at the CR-BS or its terminals, or through an external set of wireless sensing networks. We assume that the instant values of CINRs are available at every resource allocation cycle. A2: The wireless channel fading is a slow-fading channel model so that the channel conditions remain unchanged during the resource allocation cycle. This assumption is practically acceptable in two deployment scenarios: fixed/low mobility environment and high data rate communication system.

A3: The downlink phase of the CRN is during the uplink phase of neighboring primary network. The uplink and downlink schedules of primary networks are maintained and updated by the Radio Environment Map (REM) [24], [25], which are available to the CR-BS to facilitate synchronization. While this assumption is not limiting to the analysis, it reduces the resource allocation complexity due to the reduced interference possibilities caused by its resource allocation.

A4: The PU locations and their transmission power can be obtained from the REM for CR-BS to estimate g_{PU-j}.

Given the assumptions and the mathematical characterization of the CRN wireless environment, a novel resource allocation framework is developed in the following section.

III. Resource Optimization Framework

The proposed resource allocation optimization framework is formulated bearing in mind several requirements and constraints. We begin by defining these requirements and constraints, then we discuss in further details the significance of the optimization objective function (Space-Bandwidth Product). Then, we list the mathematical formulation of the resource allocation optimization framework.

A. System Requirements and Limitations

There are four system requirements addressed in this resource allocation framework. One of them is its awareness of PU activity in order to protect the PU from harmful interference [5]. In our proposed framework, this awareness is obtained in two dimension (i.e., time and space) through two measured parameters: the activity $\omega^{(k)}$of PU p at channel k, and the channel gain g_{PU_p}between CR-BS and PU. The first parameter provides the temporal information of PU activity at channel k, and it is a parameter that ranges between 0 and 1, where 0 refers to no PU activity while 1 refers to a continuous PU activity. The second parameter provides spatial information about the PU in order to evaluate the harm caused by late resource allocation adaptation to PU activity on a given channel. As a result, the following resource allocation framework can distinguish between close and far PUs, and between high and low PU activities. The second requirement is to avoid allocating the same channel to more than one user within the same sector. The third and fourth requirements are related to the CR users and they are the data rate and minimum received power requirements.

In addition to the requirements, a number of system limitations are considered such as the limited range of channels that can be accessed by the CR user. We define $\overline{\mathcal{K}_j}$ as the set of inaccessible channels for user j due to its hardware limitations. Another limitation is set by the sensitivity of CR receiver which defines the minimum received power threshold in the framework. Also, the maximum level of interference that can be sustained by PU p at channel k limits the maximum power that can be emitted by CR at that channel. This defines the maximum interference threshold $\Gamma_{th_p}^{(k)}$. Nevertheless, this interference occurs only when PU is using this channel. Otherwise, CR-BS can take full advantage of the power resource at that channel. Limiting the transmission power continuously based on PU interference threshold on any channel can be a waste of opportunities for CRN. Instead, the interference constraints on a certain channel should be valid only when the PU is active on that channel. This is obtained by the PU activity parameter. Since there is a level of confidence and reliability associated with measuring that parameter, we define an activity threshold ω_{th}, which activates the PU interference constraint in the optimization framework. This threshold can be optimized for certain hardware limitations and PU activity patterns. However, finding the optimum value of that threshold is beyond the scope of this work.

B. Space-Bandwidth Product

The space-bandwidth product, which is defined as the multiplication of the bandwidth utilization and transmission power (an alternative measurement for coverage space), is similar to the transport capacity defined in [8] to study capacity scaling laws in large wireless networks. This metric can be used to capture the efficiency of spectrum utilization. It is motivated by the fact that wireless communication consumes space [8]. In CRNs, the gain of spectrum sharing comes from the heterogeneity in space consumption of different types of wireless devices with different bandwidth [13]. In this context, a user with large bandwidth demand is allocated first to achieve better utilization. Thanks to their frequency agile features, CRNs are capable of adapting their spectral resource utilization (i.e. bandwidth and transmission power) based on the spectral activity, density and distribution of adjacent primary networks. As a result, CRNs can achieve better utilization of space (i.e., spectral footprint).

Fig. 2 illustrates the difference between the resource allocation using waterfilling and joint bandwidth-power product minimization. The waterfilling solution attempts to use as much bandwidth as possible to minimize the power. This aggressive allocation of channels in a CRN that relies on overlay spectral sharing can waste a lot of spectral opportunities. However, the spectral footprint optimization, through the bandwidth-power product, reduces the number of channels that are allocated at the expense of higher transmission power so that the overall spectral footprint is minimized. Yet, the resource allocation based on the spectral footprint minimization favors high channel gain to noise and interference channels as is the case in the waterfilling algorithm.

C. Problem Formulation

Next, we develop the mathematical formulation of the resource allocation optimization framework, which addresses all the requirements and limitations mentioned above. The problem is formulated as a spectral footprint minimization problem, where the spectral footprint is expressed by the product of the bandwidth footprint F_B and the power footprint F_P. This product is the objective function of the formulated minimization problem. Let $x_j^{(k)}$ be a binary variable, which is equal to *one* if the k-th channel is allocated for CR user j, and is equal to *zero* otherwise. Also, let $p_j^{(k)}$ be a continuous real variable that represents the power transmission of the CR-BS on channel k to user j. These two variables, which represent the resources considered in the proposed framework, are allocated under several channel allocation and power allocation constraints. The formulated minimization problem is described mathematically as follows:

$$\min\{F_B F_P\}:F_B=\sum_{j=1}^{|J|}\sum_{k=1}^{K}\frac{B}{1-\omega^{(k)}}x_j^{(k)},\ F_P=\sum_{j=1}^{|J|}\sum_{k=1}^{K}p_j^{(k)} \tag{4}$$

subject to

$$\sum_{k\in\overline{\mathcal{K}_j}} x_j^{(k)}=0,\quad \sum_{j=1}^{|J_i|}x_j^{(k)}\leq 1, \tag{5}$$

$$\sum_{j=1}^{|J|}\mathcal{I}_p^{(k)}\leq\Gamma_{th_p}^{(k)}\ \text{when}\ \omega^{(k)}\leq\omega_{th},\quad \sum_{k=1}^{K}g_j^{(k)}p_j^{(k)}\geq\Gamma_j, \tag{6}$$

$$\sum_{k=1}^{K}Bx_j^{(k)}\log\left(1+\frac{g_j^{(k)}p_j^{(k)}}{\eta+\sum_{p\in\mathcal{P}}\mathcal{J}_{pj}^{(k)}}\right)\geq\phi_j, \tag{7}$$

$$\forall x_j^{(k)}\in\{0,1\},\quad p_j^{(k)}\in\mathcal{R}^+\cup\{0\},\quad i\in\mathcal{C},$$

where Γ_j is the sensitivity limit and ϕ_j is the minimum QoS requirements in terms of average data rate. In the following, we discuss the objective function and optimization constraints.

1) Objective Function: The spectral footprint is expressed as the the product of the bandwidth footprint F_B and the power footprint F_P in (4). In order to accommodate for the PU spectral activities in the optimization process, the bandwidth footprint is adjusted by $\omega^{(k)}$. This coefficient, which is called the PU activity index, captures the impact of PU activities on the resource allocation process. This is achieved by applying a cost factor to a channel k that is proportional to the level of PU activity on that channel. When PU activity is low (close to *zero*), the cost represented by $\tilde{\omega}^{(k)}=\frac{1}{1-\omega^{(k)}}$ is low. Similarly, as PU activity approaches *one* on channel k, the cost associated with allocating this channel is high. As a result, the optimization framework aims to avoid allocating channels with high PU activities. To calculate PU activity index, each user first samples the PU activity by monitoring the spectrum band and reports the samples to CR-BS. These monitored PU activity samples collected by CR-BS are accumulated into clusters using first-difference filtering and their temporal correlation. Based on these statistics, CR-BS is able to keep track of highly dynamic changes in PU activity and calculate the PU activity index according to the method in [5]. Interested reader on the subject of PU activity modeling and estimation can refer to [5].

2) Constraints: The constraints (5)–(7) are divided into several categories: channel allocation constraints in (5), power allocation constraints in (6), and QoS constraint in (7).

The *channel allocation constraints* in (5) regulate the channel assignments at the terminal level and the macrocell level. At terminal level, the channel allocation constraint takes into considerations the hardware specification of wireless terminals. Each terminal can access one or more channels. However, the bandwidth of the terminal is limited compared to the CR-BS bandwidth. Hence, each terminal can only support a list of channels $\mathcal{K}$ that it can access. At cell level, the number of users that can utilize a particular channel within a BS range at a particular time depends on the spatial diversity techniques implemented at the BS. The spatial diversity is captured by the parameter C, which is the maximum frequency reuse factor that can be achieved (e.g. sectoring by beamforming) in the cell. This parameter is useful to model the application of frequency reuse through beamforming. In case beamforming is not implemented, $C=1$.

The *power constraints* determine the upper and lower bounds of the transmission power that can be transmitted. The upper bound is set by the first inequality in (6), which implies that the power is bounded at each channel k by the PU interference limit if its activity exceeds a certain level. The lower bound is set by the second inequality in (6) where each power spectral density allocated for the j-th user at a given channel should be higher than the sensitivity limit Γ_j.

The *QoS constraint* in (7) assures meeting the minimum QoS requirements ϕ_j by the channel allocation $x_j^{(k)}$ and power allocations $p_j^{(k)}$. The QoS is expressed as the average data rate between CR-BS and its user j.

IV. Optimized Resource Allocation Solution Using Decomposition Theory

The optimization variables to be determined in the formulated resource allocation optimization problem are the channel

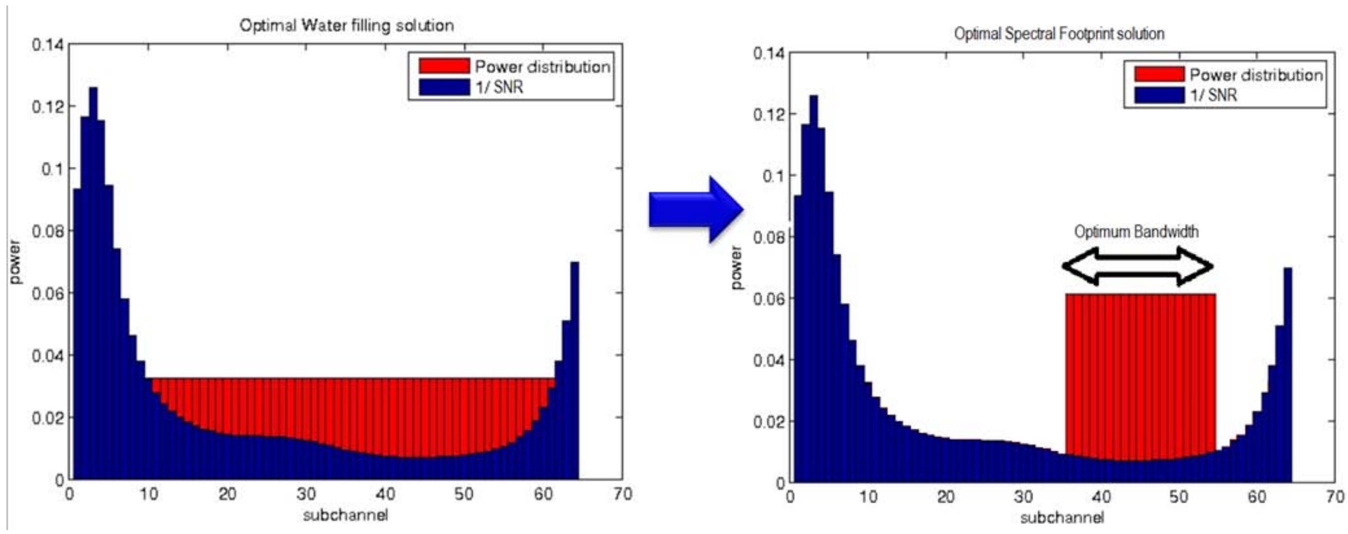


Fig. 2. Resource allocation using waterfilling and bandwidth-power product minimization.

allocation x (a binary variable) and transmission power p (a continuous variable). Thus, the formulated optimization problem is a mixed integer non-linear programming problem, which is NP-hard to solve in general. The computational complexity becomes prohibitively high for large-sized networks. In the following, we propose solving this problem using iterative methods based on *decomposition theory* [17], [18].

The overall optimization problem is a joint channel-power allocation problem. The problem is decomposed into a master problem and a sub-problem via vertical decomposition. The master problem is channel allocation based on a predetermined power budget, while the sub-problem is a power allocation for the channel allocation found by solving the master problem. In addition, a horizontal decomposition is conducted on the channel allocation and power allocation problems. The vertical decomposition, or sometimes referred to as primal decomposition [17], [18] is achieved by eliminating the impact of the coupling optimization variables. While the horizontal decomposition, also known as dual decomposition [17], [18], is achieved by eliminating the impact of the coupling constraints. In the following sub-sections, we examine a simplified joint bandwidth-power allocation for a single user in order to evaluate the impact of the new metric (i.e. the bandwidth-power product) on the resource allocation outcome. Hence, no power or channel activity constraints are considered in the single user scenario. Then, we report in details of the resource allocation solution for the multiuser case.

A. Single User Allocation

In this subsection, we analyze the resource allocation problem for a single user based on spectral footprint minimization. Let us consider a set of N channels that are sorted in descending order according to their CINR values measured at the receiver. By sorting the channels, we can express the channel allocation in terms of the number of allocated channels K starting from the first channel instead of the binary variable x. The bandwidth of each channel is B, and the PU activity index is $\omega^{(k)}$ where k is the channel index that ranges from 1 to N. As a result, the total number of channels $K \in \{1, \ldots, N\}$ and the power $p^{(k)} \in \mathcal{R}^+ \cup \{0\}$ become the resource allocation variables to be determined for the user in order to meet its data rate requirements ϕ. The following resource allocation optimization problem can be formulated as follows:

$$\min_{K,p}\{F_B F_P\} : F_B = \sum_{k=1}^{K} B\tilde{\omega}^{(k)}, F_P = \sum_{k=1}^{K} p^{(k)}, \quad (8)$$

subject to

$$\sum_{k=1}^{K} B \log\left(1 + \frac{g^{(k)} p^{(k)}}{\eta + \sum_{p\in\mathcal{P}} \mathcal{J}_p^{(k)}}\right) \geq \phi. \quad (9)$$

This problem can be solved in two phases. First, we determine the power for a given number of allocated channels K^*, then we determine the optimal K value after the power allocation is found to minimize the bandwidth-power product.

1) Power allocation:

$$\min_{p}\{F_B^* F_P\} : F_B^* = \sum_{k=1}^{K^*} B\tilde{\omega}^{(k)}, F_P = \sum_{k=1}^{K^*} p^{(k)}, \quad (10)$$

subject to

$$R^* = \sum_{k=1}^{K^*} B \log\left(1 + \frac{g^{(k)} p^{(k)}}{\eta + \sum_{p\in\mathcal{P}} \mathcal{J}_p^{(k)}}\right) \geq \phi. \quad (11)$$

This problem can be solved by formulating the Lagrangian function as follows:

$$L = F_B^* F_P + \lambda(\phi - R^*) \quad (12)$$

where $\lambda \geq 0$ is a Lagrangian multiplier. By minimizing L as a function of $p^{(k)}$. The solution of this Lagrangian dual problem becomes:

$$p^{(k)} = \left[\frac{2^{\frac{\phi}{K^* B}}}{\prod_{K^*}(h^{(k)})^{1/K^*}} - \frac{1}{h^{(k)}}\right]^+ \quad (13)$$

where $h^{(k)}$ is defined by (3), and $[x]^+$ is the projection of x into the non-negative area. As we can see the power allocation solution is a form of the waterfilling solutions. However, the range of channels over which the allocation is conducted is bounded by K^*.

2) Channel allocation: After obtaining the optimum power setting for a pre-defined number of allocated channels K, we find K that minimizes the bandwidth-power product:

$$\min_{K}\left\{\left(\sum_{k=1}^{K} B\tilde{\omega}^{(k)}\right)\left(\sum_{k=1}^{K}\frac{2^{\frac{\phi}{KB}}}{\prod_{K}(h^{(k)})^{1/K}}-\frac{1}{h^{(k)}}\right)\right\}. \quad (14)$$

This problem can be solved iteratively using Gauss–Newton method [4]. Assuming that $\overline{h}$ is the average $h^{(k)}$ across all channels $k \in \{1, 2, \ldots K\}$, and $\Delta^{(k)}$ is the difference between the instant value of CINR $h^{(k)}$ and its average $\overline{h}$, we have $h^{(k)} = \overline{h} + \Delta^{(k)}$. As a result, (14) becomes:

$$\min_{K}\left(\sum_{k=1}^{K}\tilde{\omega}^{(k)}\right)\left(\frac{K2^{\frac{\phi}{KB}}}{\prod_{K}(\overline{h}+\Delta^{(k)})^{\frac{1}{K}}}-\sum_{k=1}^{K}\frac{1}{\overline{h}+\Delta^{(k)}}\right). \quad (15)$$

We note that the optimal number of allocated channels K does not depend directly on CINR values of each channel. Instead, it depends on the fluctuations (collective view) of the CINR over all the channels as can be seen from the summation and product terms. Under the special case where the variations of the CINR over all the candidate channels are very small, $\Delta^{(k)} \ll \overline{h}$, and the PU activity across different channels are relatively comparable $\omega^{(k)} \approx \omega$, we obtain $K = \frac{\phi}{2.3B}$. This special case is possible when the coherence bandwidth is larger than the bandwidth of K allocated channels. It is more likely to encounter this case in indoor communication where the coherence bandwidth is generally larger than the one for outdoor communication [9].

To compare the performance of the proposed algorithm and the waterfilling algorithm analytically using the bandwidth-power product metric, we need to estimate the number of allocated channels and power level. For power allocation, the power level allocated by the proposed algorithm, $p^{(k)}$, is given in (13) while the power level allocated by waterfilling is defined as

$$p^{(k)} = \left(\frac{1}{\lambda} - \frac{N_0}{h^{(k)}}\right) \quad (16)$$

where $\frac{1}{\lambda}$ is the water level, $h^{(k)}$ is the channel gain, and N_0 is the power spectral density of the Gaussian noise in AWGN channel. For channel allocation, the estimation of the number of allocated channel is nontrivial. In very special cases where the variation of CINR and PU activities are very small across different allocation channels, the number of allocated channels using our proposed allocation algorithm can be estimated as $K = \frac{\phi}{2.3B}$. The number of channels allocated by the waterfilling algorithm, $K^{'}$, on the other hand, is the number of channels under the water level given by $K^{'} = \sum_{k} I(h_j^{(k)} \geq \lambda)$, where $h_j^{(k)}$ is the CINR at CR user j in channel k and $I(x)$ is the indicator function that is 1 if x is true and 0 otherwise. Thus, the bandwidth-power product of the proposed algorithm is $BP(\text{proposed}) = \frac{\phi}{2.3}\cdot\sum_{k=1}^{K} p^{(k)}$, where $p^{(k)}$ is given in (13), while the bandwidth-power product of the waterfilling case is $BP(\text{waterfilling}) = K^{'}B \cdot \sum_{k=1}^{K^{'}} p^{(k)}$, where $p^{(k)}$ is defined in (16).

In light of the study above, we address the multiuser resource allocation problem in the next subsection.

B. Multiuser Resource Allocation

In the single-user resource allocation case, a greedy approach is adopted, where the channels with highest CINR are assigned first because they require the least additional power. This approach leads to the optimal allocation in the sense of minimizing the overall spectral footprint. Unfortunately, the problem becomes more difficult in the case of multiuser resource allocation because different users experience different levels of CINR at each channel. Orthogonal resource allocation based on Karhunen-Loève expansion is no longer applicable since there is no kernel that simultaneously diagonalizes the channel responses of all the users [12]. In this work, we assume the full knowledge of the channel CINR values at the CR-BS. This is a common assumption in literature [3]. We solve the optimization problem presented in Section III using decomposition theory. Primal (vertical) decomposition is applied to solve the single user case in Section IV-A. However, in the case of multiuser resource allocation, multiple decompositions are used to obtain the allocation solution.

The vertical decomposition is performed on an optimization problem that has a coupling variable such that, when fixed to a certain value, the rest of the optimization problem decouples into a master problem and one or more subproblems. The master problem resulted from the vertical decomposition can be interpreted as the problem of distributing resources to the subproblems. In our case, the joint bandwidth-power allocation problem is decomposed into bandwidth allocation and channel assignment problem (master problem) and a power allocation problem (sub-problem). The coupling variables are the power allocation variables, and we aim to fix them at the beginning to feasible values. In fact, it is sufficient to fix the total power allocation in order to achieve the required decoupling. Then, the channel allocation (master problem) is performed in two steps. First, we estimate the total bandwidth of K^* allocated channels required to support all users based on the data rate requirements and channel conditions. Second, we determine the channel assignment $[K_1 \ldots K_{|J|}]$ for each user based on the estimated bandwidth budget. Upon determining the channel allocation, the power allocation is determined accordingly (sub-problem). At this point, a complete cycle of channel allocation followed by power allocation is completed. This allocation cycle iterates by evaluating the drop of consumed bandwidth-power product. If this drop is not small enough to stop the iterative process, a new total bandwidth budget K^* is estimated. The power allocation takes into account the last channel allocation obtained from the previous cycle. Fig. 3 shows the iterative channel and power allocation process.

1) Master Problem–Total Bandwidth Allocation and Channel Assignment: At the beginning, we fix the transmission power to a feasible value $p_j^{*(k)}$. As a result, we obtain the following optimization problem.

$$\min \sum_{j=1}^{|J|}\sum_{k=1}^{K}\frac{B}{1-\omega^{(k)}}x_j^{(k)} \quad (17)$$

subject to

$$\sum_{k\in\overline{\mathcal{K}_j}} x_j^{(k)} = 0, \forall x_j^{(k)} \in \{0,1\}, \quad \sum_{j=1}^{|J_i|} x_j^{(k)} \leq 1, \forall i \in \mathcal{C} \quad (18)$$

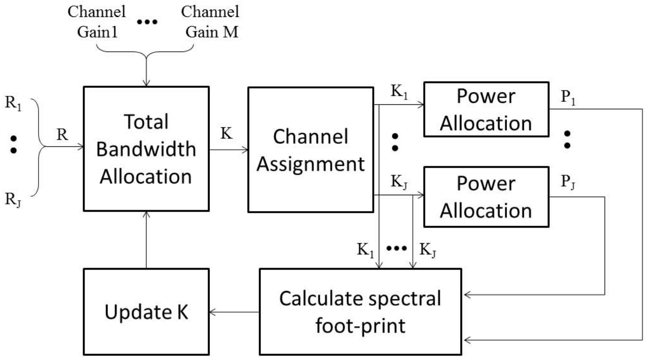


Fig. 3. Resource allocation process.

$$\sum_{k=1}^{K} Bx_j^{(k)} \log\left(1+\frac{g_j^{(k)}p_j^{*(k)}}{\eta+\sum_{p\in\mathcal{P}}\mathcal{J}_{pj}^{(k)}}\right) \geq \phi_j. \tag{19}$$

This is a binary integer programming problem. The objective in this problem is to minimize the total consumed bandwidth. This minimization is practically bounded by the data rate requirement constraints and PU activities. We propose a novel method for solving this problem based on a modified version of the Hungarian assignment algorithm [15]. The proposed method is performed in two stages.

In the first stage, the total number of channels required for all users (K^*) is estimated. We refer to this number as the bandwidth budget. This number can be estimated by summing the bandwidth budget requirement (K_j) for each user j as per (14) based on its data rate requirement and observed CINR values. The bandwidth budgets $\{K_j\}$ for all users are passed along with other parameters to the channel assignment algorithm as illustrated in Algorithm 1.

In the second stage, the cost matrix for channel assignment is formulated to assign K^* channels to $|J|$ users. The Hungarian method is the assignment algorithm used for this purpose. Since the user sets $\{J_i\}$ are orthogonal, the channel assignment can be performed independently for each cell sector. In other words, the assignment table is formulated for K^* channels and $|J_i|$ users. Clearly, if $|J_i| > K^*$, the assignment problem is infeasible. Otherwise, the assignment algorithm proceeds as follows: We construct a $|J_i| \times K$ cost matrix $\mathbf{C}$. The elements of this matrix $c_j^{(k)}$ represents the cost of assigning channel k to user j. The cost associated with each channel assignment is based on the PU activity at that channel and the gain value observed by the user at that channel. The cost is calculated as follows:

$$c_j^{(k)} = \frac{\eta+\sum_{p\in\mathcal{P}}\mathcal{J}_{pj}^{(k)}}{(1-\omega^{(k)})Bg_j^{(k)}}. \tag{20}$$

In order to incorporate the constraint (18) into the assignment algorithm, we set the channels that do not belong to the supported sets of users to an arbitrary high value. After obtaining the cost matrix, we obtain the best $K^* \leq K$ channels for each user sorted based on the cost value. Although we need only K_j channel for each user, we identify the top K^* to account for the overlap that might occur between the lists of best K_j channels for each user. As can be seen in Algorithm 1, an index matrix $\mathbf{D}$ is obtained by the sort function. This matrix stores the index of the top K^* channels in each user. The algorithm then process by checking the conflict between the top K_j channel assignments among users. If a channel k for user j is not found in the top K_j list of other users $\mathbf{R}$, the channel is assigned to user j and is removed from $\mathbf{D}$ matrix. As a result the channel budget K_j for user j is reduced by one.

Algorithm 1 : Channel Allocation Algorithm

1: **Input:** $\{K_j\}, \overline{\mathcal{K}}, \{\omega^{(k)}\}, \{g_j^{(k)}\}, \{J_i\}, C$
2: **Output:** $\{x_j^{(k)}\}^*$
3: **for** $i \leftarrow 1$ to C **do**
4: $\quad K^* \leftarrow \sum_{j=1}^{|J|} K_j$
5: $\quad$ **if** $|J_i| > K^*$ **then** $\triangleright$ Infeasible Assignment
6: $\quad\quad$ **return** $\{x_j^{(k)}\}^* \leftarrow \{0\}$
7: $\quad$ **else**
8: $\quad\quad$ **for all** $j \in J_i$ **do**
9: $\quad\quad\quad$ **for** $k \leftarrow 1$ to K **do**
10: $\quad\quad\quad\quad$ **if** $k \in \overline{\mathcal{K}_j}$ **then**
11: $\quad\quad\quad\quad\quad$ $\mathbf{C}[j,k] \leftarrow \infty$
12: $\quad\quad\quad\quad$ **else**
13: $\quad\quad\quad\quad\quad$ $\mathbf{C}[j,k] \leftarrow \frac{\eta+\sum_{p\in\mathcal{P}}\mathcal{J}_{pj}^{(k)}}{(1-\omega^{(k)})Bg_j^{(k)}}$
14: $\quad\quad\quad\quad$ **end if**
15: $\quad\quad\quad$ **end for**
16: $\quad\quad\quad$ $[\mathbf{C}_{sorted}, \mathbf{D}] \leftarrow sort(\mathbf{C}, K^*, \text{'Descending'})$
17: $\quad\quad$ **end for**
18: $\quad\quad$ **for all** $j \in J_i$ **do**
19: $\quad\quad\quad$ $\mathbf{R} \leftarrow \mathbf{D}$[all users except j , all k]
20: $\quad\quad\quad$ **for** $k \leftarrow 1$ to K_j **do**
21: $\quad\quad\quad\quad$ **if** $k \notin \mathbf{R}$ **then**
22: $\quad\quad\quad\quad\quad$ $x_j^{(k)} \leftarrow 1, \quad K_j \leftarrow K_j - 1$
23: $\quad\quad\quad\quad\quad$ remove all k from $\mathbf{D}$
24: $\quad\quad\quad\quad$ **end if**
25: $\quad\quad\quad$ **end for**
26: $\quad\quad$ **end for**
27: $\quad\quad$ **if** $\sum_{j=1}^{|J_i|} K_j \neq 0$ **then**
28: $\quad\quad\quad$ $\mathbf{T} \leftarrow \mathbf{C}[\forall j, \forall k \in \mathbf{D}]$
29: $\quad\quad\quad$ Duplicate the row of each user j by K_j
30: $\quad\quad\quad$ $\{x_j^{(k)}\}^* \leftarrow Hungarian\_Method(\mathbf{T})$
31: $\quad\quad$ **end if**
32: $\quad$ **end if**
33: **end for**

The best case scenario occurs when there is no overlap between the top K_j channels for each user. In this case, the channel allocation is done, and these channels are allocated to their associated users. However, if there is an overlap between the best K_j channels for each user, then this overlap should be resolved using the assignment table and Hungarian algorithm [15]. If there is any remaining channel budget $K_j \neq 0$, we formulate the assignment table $\mathbf{T}$. The columns of this table are the channels left in the $\mathbf{D}$ matrix. The table rows represent the list of users. Each user row is repeated K_j times. If $K_j = 0$, the row associated with user j is removed from the assignment table because its channel allocation requirements is already fulfilled by the previous loop in lines (18-27). The Hungarian assignment algorithm is then applied on the resulting assignment table to obtain the rest of channel allocations $\{x_j^{(k)}\}$.

2) Sub-problem–Power Allocation: Based on the allocated channels found in the master problem, the power between the CR-BS and its associated users j at each channel k is allocated by solving the following optimization problem:

$$\min F_B^* \sum_{j=1}^{|J|} \sum_{k \in \mathcal{K}_j} p_j^{(k)} \tag{21}$$

subject to

$$\sum_{j=1}^{|J|} \mathcal{I}_p^{(k)} \leq \Gamma_{th_p}^{(k)} \text{ when } \omega^{(k)} \leq \omega_{th}, \sum_{k \in \mathcal{K}_j} g_j^{(k)} p_j^{(k)} \geq \Gamma_j, \tag{22}$$

$$R_j = \sum_{k \in \mathcal{K}_j} B \log\left(1 + \frac{g_j^{(k)} p_j^{(k)}}{\eta + \sum_{p \in \mathcal{P}} \mathcal{J}_{pj}^{(k)}}\right) \geq \phi_j. \tag{23}$$

Note that we have omitted the binary variable $x_j^{(k)}$ from (23) since the summation is done over the range of allocated channels $\mathcal{K}_j$. This problem is solved using dual-decomposition method, where the Lagrangian variables are calculated in distributive fashion using iterative sub-gradient algorithm [4]. The Lagrangian variables are then used to obtain the optimum transmission power values. We can formulate the Lagrangian function as (24), where λ, γ, and μ are non-negative Lagrangian multiplier vectors. We aim to optimize the power setting as follows:

$$p_j^{*(k)} = \arg \min_{p_j^{(k)} \geq 0} L(p, \lambda, \gamma, \mu). \tag{25}$$

For each optimization problem, the following equations are derived:

$$\frac{\partial L(p, \lambda, \gamma, \mu)}{\partial p_j^{(k)}} = 0, \tag{26}$$

$$\lambda_p^{(k)}\left(\sum_{j=1}^{|J|} p_j^{(k)} I_p^{(k)} - \Gamma_{th_p}^{(k)}\right) = 0, \gamma_j\left(\Gamma_j - \sum_{k \in \mathcal{K}_j} g_j^{(k)} p_j^{(k)}\right) = 0,$$

$$\mu_j\left(\phi_j - \sum_{k \in \mathcal{K}_j} B \log\left(1 + \frac{g_j^{(k)} p_j^{(k)}}{\eta + \sum_{p \in \mathcal{P}} \mathcal{J}_{pj}^{(k)}}\right)\right) = 0. \tag{27}$$

Solving these equations is done iteratively as follows. First, we find the transmission power as a function of the Lagrangian multipliers λ, γ, and μ from (26):

$$p_j^{(k)} = \left[\frac{\mu_j B}{F_B^* + \lambda_p^{(k)} - \gamma_j g_j^{(k)}} - \frac{\eta + \sum_{p \in \mathcal{P}} \mathcal{J}_{pj}^{(k)}}{g_j^{(k)}}\right]. \tag{28}$$

Several observations can be made from (28) about the relation between the allocated power on the active power allocation constraints. If the data rate constraint is violated for an allocated power value $p_j^{(k)}$, the associated Lagrangian multiplier μ_j increases leading to an increase in the allocated power. Similarly, the allocated power increases by increasing the value of γ_j when the minimum power constraint (i.e., sensitivity constraint) is violated. However, when the PU interference constraint is violated, the associated Lagrangian multiplier λ increases. This reduces the allocated power. Also, the more bandwidth budget F_B^* is allocated, the less the value of $p_j^{(k)}$.

Algorithm 2 : Power Control Algorithm

Input: $\{x_j^{(k)}\}^*, \{\mathcal{K}_j\}, \{\phi_j\}, \{\omega^{(k)}\}, \{g_j^{(k)}\}, \mathcal{P}, C, \{g_{PU_p}^{(k)}\}$
Output: $\{p_j^{(k)}\}^*$
Initialization: $n \leftarrow 0, \mu_j(0) \leftarrow \phi_j, \gamma_j(0) \leftarrow 0,$ $\lambda_p^{(k)}(0) \leftarrow 0, p_j^{(k)}(0) \leftarrow 0$
while $\Delta_p \geq \Delta_{th}$ **do**
  for $j \leftarrow 1$ to $|J|$ **do**
    for each $k \in \mathcal{K}_j$ **do**
      $p_{j_{old}}^{(k)} \leftarrow p_j^{(k)}$
      update $p_j^{(k)}$ as per (28)
      $\Delta_p^{(k)} = |p_j^{(k)} - p_{j_{old}}^{(k)}|$
      for each $p \in \mathcal{P}$ **do**
        update $\lambda_p^{(k)}$ as per (29)
      end for
    end for
    update γ_j as per (30) and μ_j as per (31)
  end for
  $n \leftarrow n + 1$
  $\alpha(n+1) \leftarrow \alpha(0)/n^3$
end while

Based on (27), we find the values of $\lambda_p^{(k)}$, γ_j, and μ_j iteratively as follows:

$$\lambda_p^{(k)}(n+1) \leftarrow \left[\lambda_p^{(k)}(n) + \alpha(n)\left(\sum_{j=1}^{|J|} p_j^{(k)} I_p^{(k)} - \Gamma_{th_p}^{(k)}\right)\right]^+ \tag{29}$$

$$\gamma_j(n+1) \leftarrow \left[\gamma_j(n) + \alpha(n)\left(\Gamma_j - \sum_{k \in \mathcal{K}_j} g_j^{(k)} p_j^{(k)}\right)\right]^+ \tag{30}$$

$$\mu_j(n+1) \leftarrow \left[\mu_j(n) + \alpha(n)\left(\phi_j - R_j\right)\right]^+ \tag{31}$$

where $[x]^+$ is the projection of x into the non-negative area, $\alpha(n)$ is the update step size. To assure convergence with time, we use a diminishing step size $\alpha(n)$ so that $\alpha(n) \geq 0, \lim_{n \to \infty} \alpha(n) = 0$, and $\sum_{n=1}^{\infty} \alpha(n) = \infty$. In our work the step size is updated as follows: $\alpha(n) = \alpha(0)/n^3$, $n = 1, 2, \ldots$. After each iteration, (28) is updated with the new values of $\lambda_p^{(k)}$, γ_j, and μ_j. The iterations stop when the difference between current value and updated value for each of the following variables: $p_j^{(k)}, \lambda_p^{(k)}$, γ_j, and μ_j is below a certain threshold (stopping criterion Δ_{th}). We have selected the initial values of the Lagrangian multipliers as follows: $\lambda_p^{(k)}(0) = 0$, $\gamma_j(0) = 0$ and $\mu_j(0) = \phi_j$. A pseudo code of the power control algorithm is listed in Algorithm 2.

By executing the channel allocation algorithm (Algorithm 1) and the power allocation algorithm (Algorithm 2), one iteration of the overall resource allocation algorithm is accomplished. The bandwidth-power product $F_B F_P$ is then calculated, and a new bandwidth budget K is allocated based on the Gradient Descent algorithm [4] to minimize this product (i.e. $F_B F_P$). The iterative allocation process continues until the change in $F_B F_P$ is below a given stopping criterion. The complete resource allocation algorithm is listed in Algorithm 3.

V. Simulation Results

In this section, we evaluate the performance and convergence of the proposed resource allocation framework. First,

$$L(p,\lambda,\gamma,\mu) = \sum_{j=1}^{|J|}\left(F_B^*\sum_{k\in\mathcal{K}_j} p_j^{(k)} + \gamma_j\left(\sum_{k\in\mathcal{K}_j}\Gamma_j - g_j^{(k)}p_j^{(k)}\right) + \mu_j\left(\phi_j - R_j\right)\right) + \sum_{p\in\mathcal{P}}\sum_{k\in\mathcal{K}_j}\lambda_p^{(k)}\left(\sum_{j=1}^{|J|} p_j^{(k)} I_p^{(k)} - \Gamma_{th_p}^{(k)}\right) \quad (24)$$

Algorithm 3 : Complete Resource Allocation Algorithm

Input: $\mathcal{Y} = \{\overline{\mathcal{K}}, \{\omega^{(k)}\}, \{g_{PU_p}^{(k)}\}, \{g_j^{(k)}\}, \{J_i\}, \{\phi_j\}\}$
Output: $\{x_j^{(k)}\}, \{p_j^{(k)}\}$
Initialization:
  $\{K_j\} \leftarrow$ *ChannelBudgetEstimation*$(\{\phi_j\}, \{g_j^{(k)}\})$
while $\Delta_{BP} \geq Tol$ **do**
  $\{x_j^{(k)}\}^* \leftarrow$ *ChannelAllocation*$(\{K_j\}, \mathcal{Y})$
  $\{\mathcal{K}_j\} \leftarrow$ *AssignChannelList*(user j)
  $\{p_j^{(k)}\}^* \leftarrow$ *PowerAllocation*$(\{x_j^{(k)}\}^*, \{\mathcal{K}_j\}, \mathcal{Y})$
  $BP \leftarrow (\sum_J \sum_K \omega^{(k)} B x_j^{(k)})(\sum_J \sum_K p_j^{(k)})$
  $K \leftarrow$ *GradientDescent*(BP, K)
  $\{K_j\} \leftarrow update(K, \{K_j\})$
end while
$\{x_j^{(k)}\} \leftarrow \{x_j^{(k)}\}^*$ and $\{p_j^{(k)}\} \leftarrow \{p_j^{(k)}\}^*$

we investigate the performance of the framework for each individual user to validate the theoretical results. The impact of system parameter settings on the allocation results is also analyzed. Second, we demonstrate that the proposed resource allocation framework outperforms the well-established margin-adaptive waterfilling algorithms in terms of consumed spectral footprint and we study the convergence of the resource allocation algorithm for multiple users. Third, we analyze the impact of several system parameters, such as the required allocated data rate for each user, the number of users and number of sectors, on the bandwidth-power product utilization using our proposed algorithm and the waterfilling algorithm.

In the simulation, we consider both single-user and multi-user resource allocation cases. The wireless channel model used in our simulation study is Rayleigh fading channel with Doppler shift $f_D = 6\,\mathrm{Hz}$. The bandwidth of each channel is $15\,\mathrm{kHz}$. The minimum received power for CR users is $-100\,\mathrm{dBm}$. The maximum interference threshold for PU is calculated based on minimum received SNR of $15\,\mathrm{dB}$. The average background noise power is $-110\,\mathrm{dBm}$. The CR users are placed randomly on a distance of $100 - 1000\,\mathrm{m}$ from CR-BS. The simulation results shown in this section are the average of the results of 1000 iterations by Monte-Carlo simulation.

A. Single User Resource Allocation

In the single-user case, we compare the efficiency of different resource utilization metrics in allocating channel and power resources to fulfill different data rates and under different wireless channel conditions. The metrics are (i) power, (ii) summation of power and bandwidth, and (iii) the product of bandwidth and power. The first metric is used as a benchmark since it captures the behavior of widely used margin-adaptive power allocation based on waterfilling technique. The second metric applies equal "importance" to power and channel resources. In this summation, the power is normalized to the maximum user power which is required to achieve $500\,\mathrm{kbps}$ using a single channel. Also, the channel allocation in the summation is normalized to total number of available channels. The third metric is the one used in our resource allocation framework and it captures the utilized spectral footprint. The number of channels considered for this case is $K = 8$, and the rate requirements for CR users vary from $100\,\mathrm{bps}$ to $500\,\mathrm{kbps}$. In this study, two channel models are considered: the first is frequency-flat channel (identical SNR for all channels) and the second is frequency-selective channel (varying SNR for each channel).

Fig. 4 shows the allocated channels, power, and total bandwidth-power product values obtained for a single user under different data rate requirements and flat fading wireless channel. The SNR values of all channels are $10\,\mathrm{dB}$. Under such wireless channel conditions, the resource optimization based on power minimization tends to consume all available channels. This is the expected behavior of waterfilling power allocation mechanism. The bandwidth-power product metric consumes less number of channels compared to the power metric or the power-bandwidth summation metrics at the expense of slight increase in the transmission power. However, the behavior of the resource allocation under different metrics converges under high data rate requirements because all available channels are consumed. In such case, the only degree of freedom available for the resource allocation is transmission power. In Fig. 4(c), the normalized bandwidth-power summation overlaps with the power minimization when the requested data rate is higher than $80\,\mathrm{kbps}$. This is because the total number of allocated channels are equal to the number of channels allocated using the power minimization algorithm as shown in Fig. 4(a). Similarly, the bandwidth-power product metric overlaps the power minimization when the requested data rate is higher than $250\,\mathrm{kbps}$ because the number of channels allocated using bandwidth-power product metric becomes equal to the number of channels allocated using the power metric. Nevertheless, the flat fading channel is the worst case scenario for the power metric in terms of the consumed channels. Under varying SNR values, the performance difference between the metrics reduces. Yet, the use of bandwidth-power product metric leads to smaller number of allocated channels. Fig. 5 pertains to the case of single user allocation with variable channel SNR values.

B. Multi-User Resource Allocation

In this case, we evaluate the performance of the proposed resource allocation framework for different number of users under different channel conditions and rate requirements. First, we demonstrated the performance of the channel allocation algorithm in the absence of PU activities as depicted in Fig. 6(a) where three CR users are considered with data rate requirements of 2, 4 and $6\,\mathrm{Mbps}$. Note that the channel allocation algorithm seeks high CINR values (low 1/CINR values in the figure). However, upon the existence of PU activities at the channel resources. The channel allocation

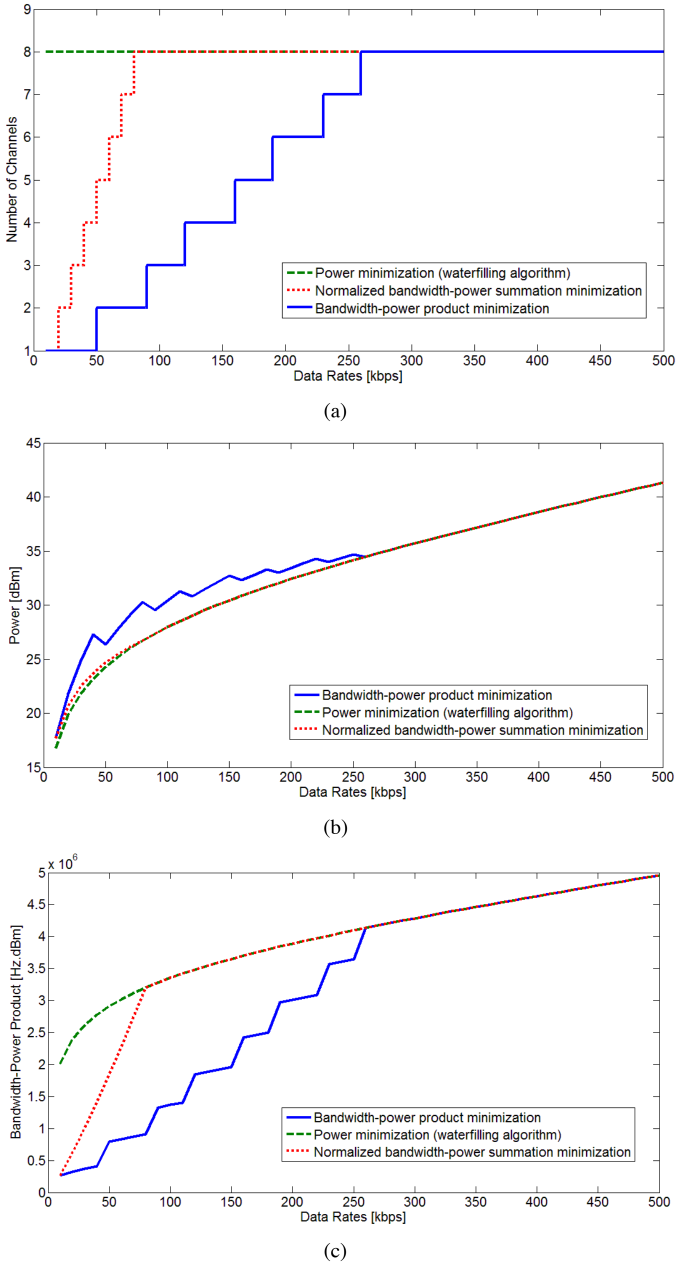


Fig. 4. (a) Number of channels, (b) transmission power, and (c) bandwidth-power product vs. data rates with SNR = 10 dB and channel bandwidth 15 kHz.

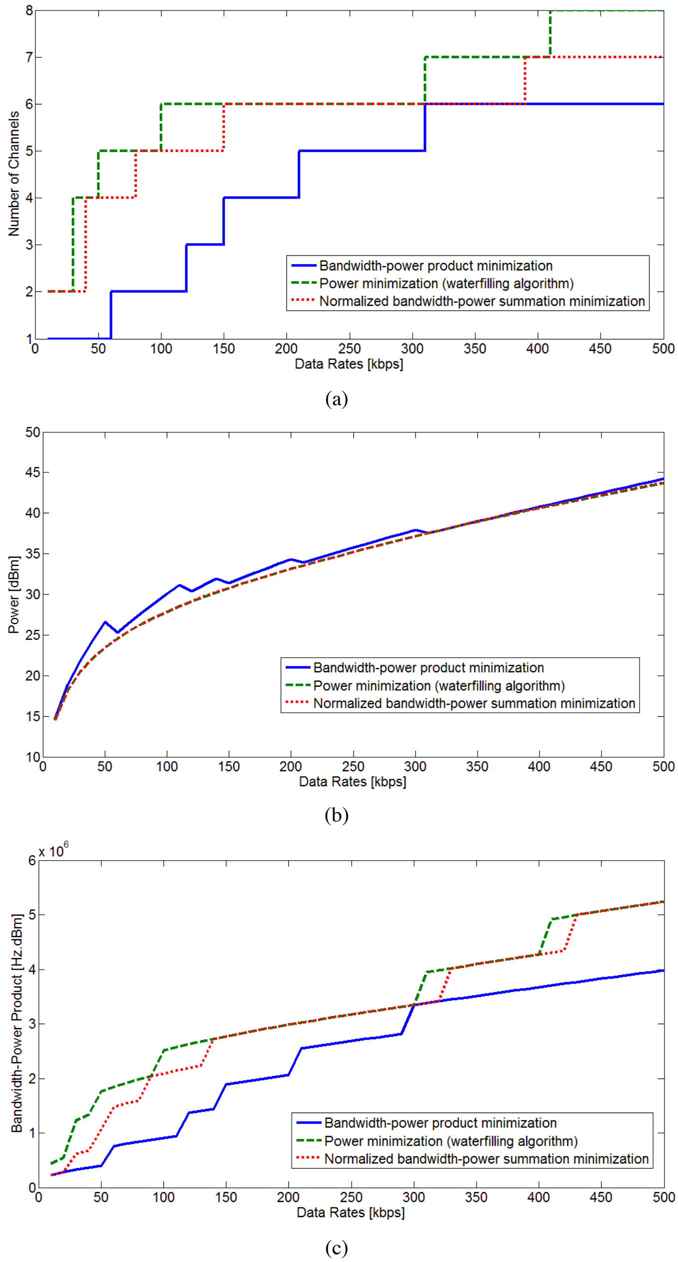


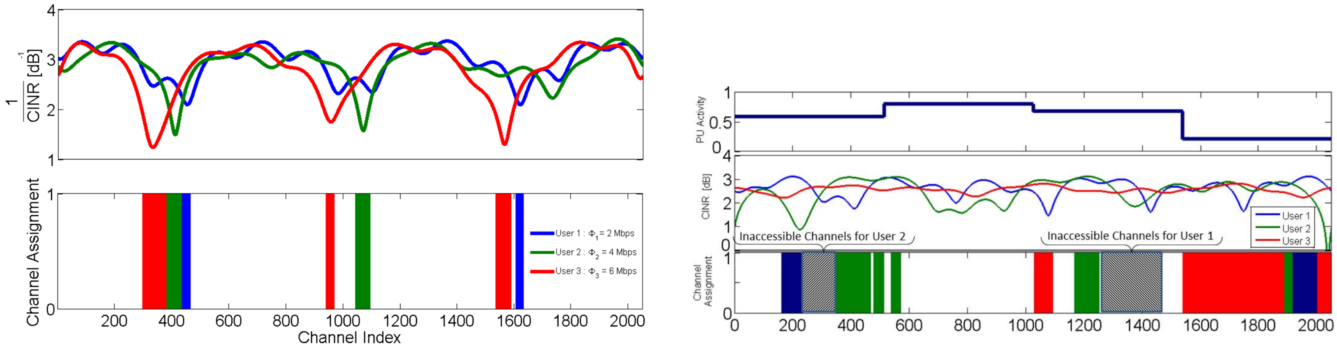
Fig. 5. (a) Number of channels, (b) transmission power, and (c) bandwidth-power product vs. data rates with SNR = [0 7 10 12 13 9 -3 10] dB and channel bandwidth 15 kHz.

algorithm avoids as much as possible assigning channels with high PU activity profile as shown in Fig. 6(b). Note also that there are inaccessible band constraints that are applied to two CR users (user 1 and 2). The avoidance of high PU activities and inaccessible bands (due to hardware limitations) are achieved through the appropriate configuration of the cost parameter associated with each channel as per (20), which scales the CINR value based on the PU activity index. In the case of inaccessible bands, the cost parameter is set to one.

In Fig. 7(a), we compare the behavior of our proposed resource allocation framework to multiuser waterfilling resource allocation technique. In this case there are three users with required data rates of 1, 2 and 3 Mbps. As shown in Fig. 7(a), the waterfilling technique allocates more channels compared to our proposed resource allocation framework, leaving much less spectral opportunities to adjacent CR networks that operate based on the overlay spectrum sharing technique. Note also that the proposed resource allocation framework avoids interfering with PU by allocating channels with low PU activity index. The convergence of the iterative bandwidth-power allocation scheme is depicted in Fig. 7(b). In each iteration, the power allocation is updated according to (28)-(31). The results show that our resource allocation scheme converges by a few iterations, e.g., around 25.

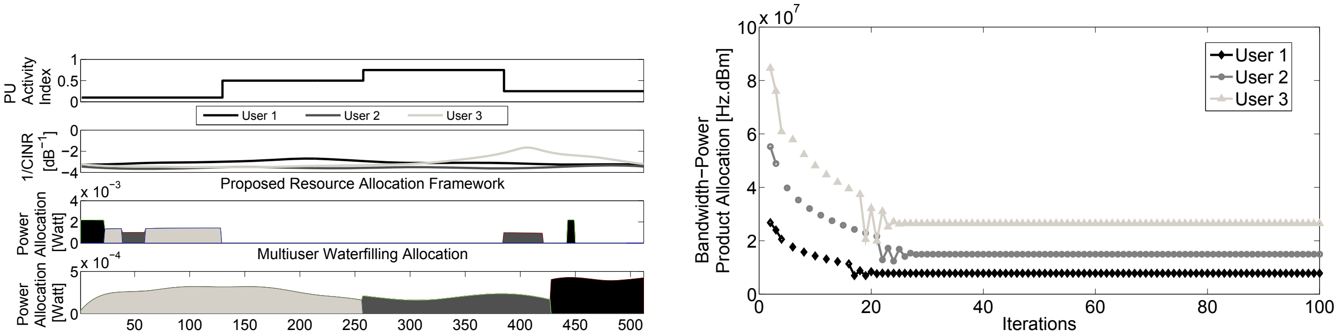
C. Spectral Efficiency and Impact of System Parameters

In this section, we analyze the impact of the following system parameters: 1) network load represented by the number of users and required data rate per user, and 2) beamforming



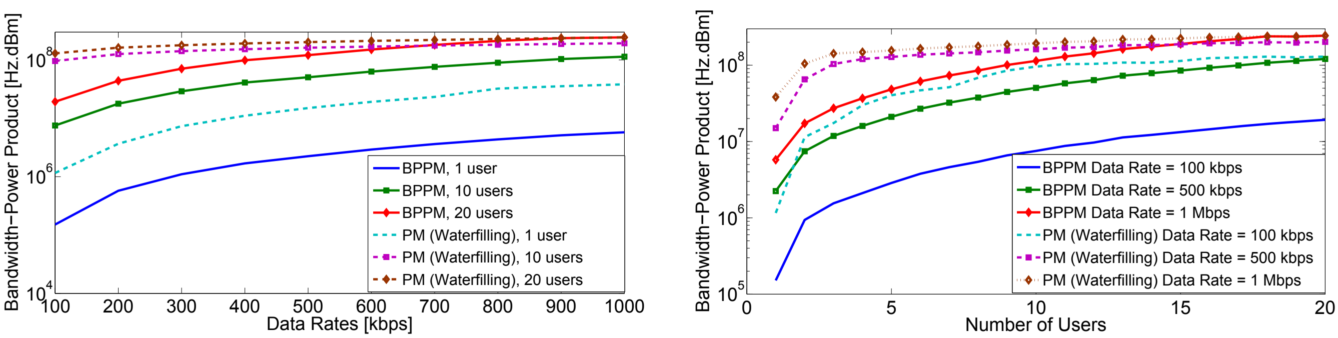
(a) (b)

Fig. 6. Channel assignment behavior with (a) no existing PU activities and (b) different PU activity levels and inaccessible bands constraints.



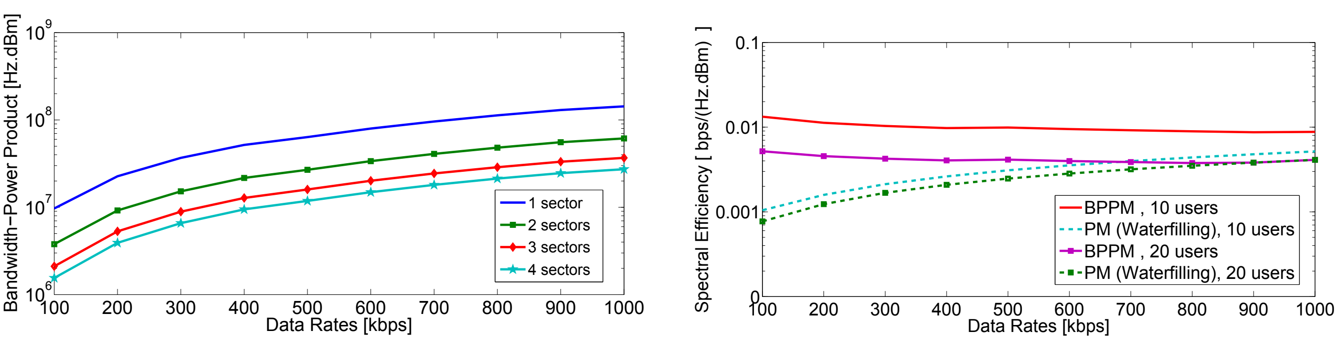
(a) (b)

Fig. 7. (a) Resource allocation behavior at different levels of PU activities and (b) convergence of bandwidth-power allocation for each CR user.



(a) (b)

Fig. 8. Bandwidth-power product utilization of the proposed BPPM algorithm and the PM algorithm at different (a) data rates for 1, 10, and 20 users, and (b) number of users for data rate = 100 kbps, 500 kbps, and 1 Mbps.



(a) (b)

Fig. 9. (a) Beamforming impact on bandwidth-power product utilization for 12 users at different user data rates using 1, 2 ,3, and 4 sectors and (b) spectral efficiency of BPPM and PM algorithms for 10 and 20 users at different user data rates.

support. We also examine the spectral efficiency achieved by the proposed algorithm compared with spectral efficiency of waterfilling algorithm. The spectral efficiency is measured by the ratio of the achieved data rate to the allocated bandwidth and power.

Fig. 8(a) shows the impact of required user data rate on the bandwidth-power product utilization using the proposed Bandwidth-Power Product Minimization (BPPM) algorithm and the Power minimization (PM) algorithm when the number of users are 1, 10, and 20 users. The bandwidth-power product utilization using BPPM is less than the utilization achieved by PM. Note that under high network load when the number of users are 20 and the required data rate per user is 1 Mbps, the bandwidth-power product utilization of BPPM and PM algorithms converge. Similarly, Fig. 8(b) shows the impact of number of users on the bandwidth-power product utilization using BPPM algorithm and PM algorithm when the required data rate per user is 100, 500, and 1000 Kbps. We see also that the reduction of bandwidth-power product utilization using BPPM as compared to the utilization of PM. The convergence of the BPPM and PM is also apparent at high network load.

The impact of beamforming is demonstrated in Fig. 9(a). As can be seen from that figure, the bandwidth-power product utilization decreases as the number of beamforms increases. However, the reduction achieved by each additional beamform diminishes as the number of supported beamforms increases. The spectral efficiency of the resource allocation can be measured using the ratio of the achieved data rates to the allocated bandwidth and power resources. Fig. 9(b) compares this efficiency achieved by BPPM and PM algorithms. We note that the BPPM spectral efficiency decreases with increasing network load, while the efficiency of PM increases to match the efficiency of BPPM at high data network load.

VI. Conclusion

We developed a new resource allocation optimization framework for a single-cell multiuser multicarrier cognitive radio network in the presence of multiple primary networks. The framework aims to minimize the spectral footprint of the CRN through the bandwidth-power product metric. The protection of PU from harmful interference is incorporated in the framework through PU activity index. In addition, several hardware limitations of CR users are considered in the proposed framework. Based on the performance evaluation results presented in this work, the proposed framework improves the utilization of spectrum by striking an optimum balance between the consumed power and bandwidth. This achievement allows for further spectral opportunities compared to what can be obtained by resource allocation based on waterfilling in CRN based on overlay spectrum sharing mechanism. The proposed framework thus provides an efficient tool for resource allocation for CRN in the presence of one or more primary networks.

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