
Reduced cluster search ML decoding for QO-STBC M-QAM MIMO systems

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Abstract: Since the maximum likelihood (ML) decoding for quasi-orthogonal space-time block codes (QO-STBC) with inner and outer codes multiple-input multiple-output (MIMO) antenna systems results too complex when the modulation order index and the number of receive antennas increases, an efficient reduced complexity ML-based decoding scheme is proposed in this contribution, aiming to reach promising QO-STBC coded MIMO system throughput \times complexity trade-off. Under high-order modulation indexes ($16 \leq M$)-QAM, $4 \times n_R$ antennas, with $n_R \geq 1$, this work proposes a reduced cluster search ML decoder (RCS-ML) and compares the performance-complexity with the ML decoding approach. Numerical results have indicated no degradation in performance and an increasing reduction in the complexity of RCS-ML decoder when the modulation order increases, been 12.5% of ML decoding complexity for 16-QAM, and <1% for 256-QAM.

Keywords: MIMO system; quasi-orthogonal space-time block codes; QO-STBC; reduced complexity ML decoding; cluster search; space-time coding; STC.

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1 Introduction

The last five years have been dominated by high demands on video and audio data with reliability real-time applications. In order to fulfill the requirements of high capacity systems, multiple-input-multiple-output (MIMO) schemes associated with space-time block codes (STBC) are frequently incorporated by many standards like WiMAX, specifically Alamouti Rate 1 STBC (Alamouti, 1998) (R1 STBC). In order to overcome the limitation of two-antennas while preserving full rate, Jafarkhani (2001) proposed QO-STBC schemes with reducing orthogonality but increasing its complexity. In Tirkkonen (2001), QO-STBCs performance were improved by using constellation rotation in order to achieve full diversity.

Furthermore, higher throughput with acceptable performance \times complexity trade-off have been achieved through the inclusion of a bit-mapped coded modulation (BMCM) structure. For instance, BMCM in conjunction with parallel short LDPC, quasi-orthogonal STBC scheme (Jafarkhani, 2005), in conjunction with iterative soft parallel interference cancellation (PIC) detector is discussed in Pau et al. (2008). The aim is to achieve relative low complexity schemes, high throughput with good performance and low processing delay in overall processing of detection and decoding. Good STBC designs must take into account the jointly optimisation of the following attributes: coding, diversity and multiplexing gains, and the decoder complexity as well.

Recently, the so called fast-decodable STBC MIMO schemes have been introduced aiming to reduce complexity with no degradation in performance. Previous works on

fast-decodable low-complexity SBTC MIMO systems includes Biglieri et al. (2008), Jiang et al. (2008), Kim and Cheun (2008), Sezginer and Sari (2007), Choi et al. (2007) and Yuen et al. (2005). In Sezginer and Sari (2007), a family of full-rate, full-diversity 2×2 space-time coding (STC), and whose detection complexity grows only quadratically with the size of the signal constellation (M) have been proposed. Thus, the optimum decoder complexity has been reduced by a factor of 256 for the 16-QAM signal constellation (and by 4,096 for the 64-QAM modulation).

A different approach to achieve low-complexity near-maximum likelihood (ML) QO-STBC decoding based on iterative interference cancellation (IICIS) was proposed in Kim and Cheun (2008). In the IICIS, the interference cancellation procedure (IC) was only used in the candidate vector pruning process and not in the detection process itself. Since the candidate pairs for decoding the transmitted symbol sub-vectors s_1 , s_4 and s_2 , s_3 in a QO-STBC scheme are updated at the end of two-consecutive IC stages, and simultaneously the number of candidate pairs decreases fast enough, few iterations are required to complete the decoding process. However, the overall decoder complexity still remains significant, especially for large modulation orders. Hence, aiming to reduce the overall decoding complexity, in Yuen et al. (2005), minimum decoding complexity (MDC) QO-STBC structures were proposed requiring the joint detection of two-real symbols regardless of the number of transmitting antennas.

Recently, an adaptive method selecting suitable MDC-QO-STBC codeword based on channel state information (CSI) has been proposed in Li et al. (2009). The CSI is used to calculate the inter-symbol interference (ISI) power of each optional MDC-QO-STBC. Then, the one with the minimum ISI power is the chosen codeword for the current transmission. For this adaptive method, it is possible to use linear decoding, achieving the lowest decoding complexity.

This contribution proposes a low-complexity efficient decoding algorithm for QO-STBCs schemes, based on reduced ML cluster search. The proposed RCS-ML decoder is suitable for any number of receive antennas, but the complexity advantage over ML decoding grows with the modulation order.

The remaining sections are divided as following. Section 2 describes the QO-STBC MIMO system. The proposed reduced cluster search ML decoding method in the context of QO-STBC codes is discussed in Section 4. Numerical results and complexity analysis for the proposed RCS-ML QO-STBC decoder with an arbitrary number of receive antennas are analysed in Section 5. Main conclusions are offered in Section 6.

2 QO-STBC system model

Let us consider a MIMO system with $n_T = 4$ transmit antennas and $n_R \geq 1$ receive antennas, where four-symbols are transmitted simultaneously; additionally, $n_T n_R$ independent subchannels are assumed. Figure 1 illustrates the main characteristics of the adopted MIMO system. In this work, non-selective fading channels, M -QAM modulation, Rate 1 QO-STBC scheme of Jafarkhani (2005) were employed.

2.1 Transmitter

The baseband transmitter is constituted by a M -QAM modulator, where each QAM symbol is mapped using $m = \log_2 M$ bits, a $1 : n_T$ demultiplexer and a quasi-orthogonal STC

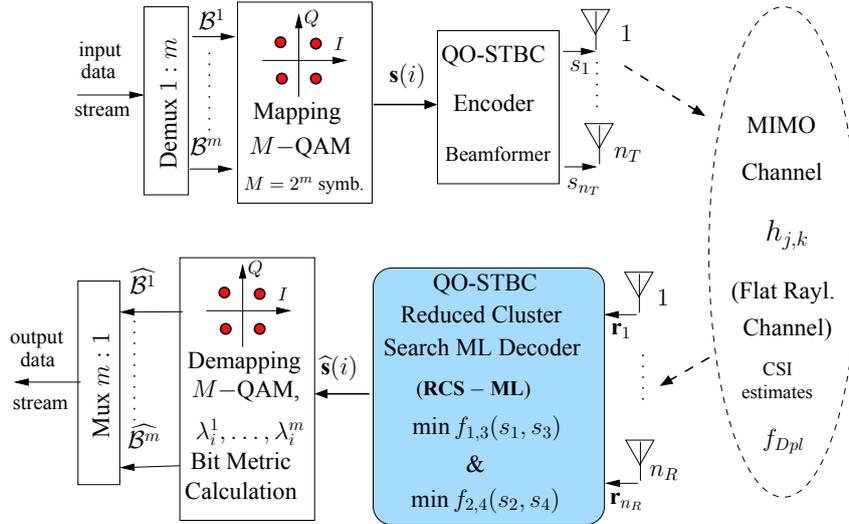
block. In order to achieve high throughput (range of 2–8 bits per symbol period, depending on the modulation index), associated to suitable performance and simplicity of decoding at the receiver, the adopted modulation is high-order squared-QAM modulation with $m = 2, 4, 6$ or 8.

The QO-STBC transmitter topology employs Rate 1 ($R_{\text{stbc}} = \frac{\text{\#symbols transmitted}}{\text{\#time slots used} \cdot L} = 1$) quasi-orthogonal space-time block code (R1 QO-STBC) as proposed in Jafarkhani (2005), and described by the code matrix:

$$\mathcal{A} = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{pmatrix},$$

where $s_i \in \mathcal{S}$ is a 2^m -ary constellation point, and \mathcal{S} is a set of all valid QAM symbols belong to the adopted constellation at the transmitter. The corresponding overall throughput for a system using a M -ary constellation is defined by: $\Theta = R_{\text{stbc}} \log_2 M = m$ [bits per symbol period].

Figure 1 QO-STBC MIMO system, suitable for high-order modulation ($M \geq 16$) with reduced cluster search ML decoding approach (RCS-ML decoding)



Note: n_T transmitted and n_R received antennas.

2.2 Receiver

Let $x(i)$ be the i th symbol modulated with duration T_s , and $s_j(t)$ the transmitted symbol by the j th transmit antenna at time t . Each transmitted symbol goes through the wireless channel to arrive at each of n_R receive antennas. Denote the path gain from transmit antenna j to receive antenna k at each symbol interval by $h_{kj}(t)$. Hence, considering a baseband

discrete-time model for a flat fading MIMO channel, the received signal at k th antenna is given by:

$$r_k(t) = \sum_{j=1}^{n_T} h_{kj}(t)s_j(t) + n_k(t), \quad t = 1, \dots, L, \quad (1)$$

where L is the time slot size of STBC block, $h_{kj}(t), \forall k \in \{1, 2, \dots, n_R\}, \forall j \in \{1, 2, \dots, n_T\}$ are assumed to be either i.i.d. complex Gaussian random variables (fade amplitudes are Rayleigh distributed) with zero mean and $\mathbb{E}[(h_{kj}^I)^2] = \mathbb{E}[(h_{kj}^Q)^2] = \frac{1}{2}$, where h_{kj}^I and h_{kj}^Q are the real and imaginary parts of $h_{kj}(t)$, or the channel matrix is assumed unitary for the case of AWGN channel. The noise sample at the k th receive antenna is assumed to be complex Gaussian with zero mean, and the samples $\{n_k\}, k = 1, \dots, n_R$ are admitted to be independents with variance:

$$\mathbb{E}[n_k^2] = N_0 = \frac{n_T \bar{E}_s}{\gamma} = \frac{n_T \bar{E}_s}{m 10^{\frac{\text{SNR}}{10}} R_{\text{stbc}}}, \quad (2)$$

where \bar{E}_s is the average energy of the transmitted symbols, given a constellation format, γ is the average received SNR per receive antenna (Jafarkhani, 2005), and SNR is the signal-to-noise ratio per receive antenna in decibels (dB).

The received signals are sampled from all receive antennas, so (1) can be rearranged in a vectorial form, and for the t th symbol period interval, the vector of the received signal is given by:

$$\mathbf{r}(t) = \mathbf{H}(t)\mathbf{s}(t) + \mathbf{n}(t), \quad t = 1, \dots, L \quad (3)$$

where for each time slot $t = 1, \dots, 4$ for the QO-STBC scheme,

$$\mathbf{r}(t) = [r_1(t) r_2(t) \dots r_{n_R}(t)]^T$$

is the received signal vector, $\mathbf{s}(t) = [s_1(t) s_2(t) \dots s_{n_T}(t)]^T$ is the transmitted symbol vector; the $n_R \times n_T$ channel matrix $\mathbf{H}(t)$, with channel coefficients h_{kj} between the j th transmitted antenna and k th receive antenna, and $\mathbf{n}(t) = [n_1(t) n_2(t) \dots n_{n_R}(t)]^T$ is the sampled noise vector. In a first analysis, the channel matrix coefficients are assumed to be known perfectly at the receiver, but completely unknown at the transmitter.

2.3 ML decoding

The ML decision metric is obtained by minimising the two-cost functions (Jafarkhani, 2001):

$$(\hat{s}_1, \hat{s}_2, \hat{s}_3, \hat{s}_4) = \arg \min_{s_1, s_4 \in \mathcal{S}} f_{1,4}(s_1, s_4), \arg \min_{s_2, s_3 \in \mathcal{S}} f_{2,3}(s_2, s_3), \quad (4)$$

where each cost function to be (independently) minimised are given by (5) and (6).

$$\begin{aligned}
 f_{1,4}(s_1, s_4) = \sum_{k=1}^{n_R} \left[\left(\sum_{j=1}^{n_T=4} |h_{jk}|^2 \right) (|s_1|^2 + |s_4|^2) + 2\Re \{ (-h_{1k}r_k^*(1) \right. \\
 - h_{2k}^*r_k(2) - h_{3k}^*r_k(3) - h_{4k}r_k^*(4))s_1 + (-h_{4k}r_k^*(1) \\
 + h_{3k}^*r_k(2) + h_{2k}^*r_k(3) - h_{1k}r_k^*(4))s_4 + (h_{1k}h_{4k}^* \\
 \left. - h_{2k}^*h_{3k} - h_{2k}h_{3k}^* + h_{1k}^*h_{4k})s_1s_4^* \} \right]
 \end{aligned} \tag{5}$$

and

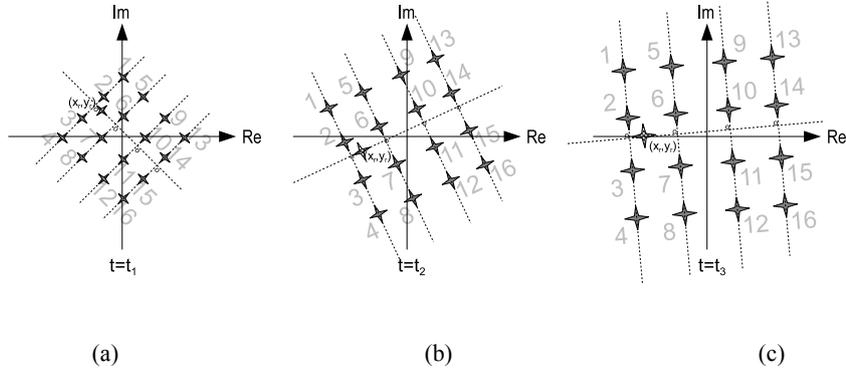
$$\begin{aligned}
 f_{2,3}(s_2, s_3) = \sum_{k=1}^{n_R} \left[\left(\sum_{j=1}^{n_T=4} |h_{jk}|^2 \right) (|s_2|^2 + |s_3|^2) + 2\Re \{ (-h_{2k}r_k^*(1) \right. \\
 + h_{1k}^*r_k(2) - h_{4k}^*r_k(3) + h_{3k}r_k^*(4))s_2 + (-h_{3k}r_k^*(1) \\
 - h_{4k}^*r_k(2) + h_{1k}^*r_k(3) - h_{2k}r_k^*(4))s_3 \\
 \left. + (h_{2k}h_{3k}^* - h_{1k}^*h_{4k} - h_{1k}h_{4k}^* + h_{2k}^*h_{3k})s_2s_3^* \} \right]
 \end{aligned} \tag{6}$$

Note that the soft estimates symbols can be interpreted as $\hat{\mathbf{s}}_i = \kappa \mathbf{s}_i + \eta_i$, where κ is the coefficient of \mathbf{s}_i in (5) or (6), and η_i is the sum of two-terms at the receiver: additive white Gaussian noise (AWGN) plus interference term due to the quasi-orthogonality of the STBC scheme.

Hence, for MIMO system with small constellation size, it is computationally viable to evaluate, independently, all possible values for the pairs (s_1, s_4) and (s_2, s_3) , using the two-cost functions (5) and (6), and directly obtain ML estimates. However, once the computation complexity increases exponentially with m , when the dimension of the constellation grows, i.e., $M \geq 16$, it is computationally inefficient to evaluate all of that pair combinations. Therefore, for high-order constellations, it becomes attractive to employ non-exhaustive procedures, or even sub-optimum methods in order to evaluate the QO-STBC cost functions. Section 3 identifies and illustrates the existence of certain pattern in (5) and (6), while Section 4 describes the proposed low complexity procedure to compute efficiently those cost functions. The procedure is specially suitable for high squared-order modulation MIMO QO-STBC with $n_T = 4$ and any number of receive antennas.

3 Pattern in cost function

Basically, the ML detector consists in determining the point in a distorted constellation closer to the received symbol testing all valid symbol. Figure 2 shows three-distinct distorted 16-QAM constellations where point 2 is the closest point to the received symbol.

Figure 2 Three typical cases of constellations distorted by the MIMO channel (modulo and phase of h_{kj}), darker stars

Note: Received symbol is the shadowed star, identified by the coordinates (x_r, y_r) .

In Figure 2(a), $t = t_1$ we have $|h| < 1$ and $\angle h < 0$, Figure 2(b) $t = t_2$ we have $|h| = 1$ and $\angle h > 0$ and Figure 2(c) $t = t_3$ we have $|h| > 1$ and $\angle h < 0$.

In order to identify patterns and clusters, we start with an analytical description to calculate the distances between received symbol and a valid symbol from the distorted constellation. Given a SISO system using a square M-QAM constellation and considering that the distance between constellation symbols is A , as shown in Figure 3, we can express the \sqrt{M} vertical lines r_p as:

$$r_p : x_p + 0 \cdot y_p = \frac{-[(\log_2 M - 1) + 2(p - 1)]}{2} A, \quad p = 1, \dots, \sqrt{M}. \quad (7)$$

Each received symbol affected by the channel is scaled by the factor $\|h\|$ and rotated by $\angle h$. But, to simplify the analysis, it will be applied the inverse effect caused by the channel to the received symbol; hence, the relative distances remain the same. In doing so, the received symbol $(\check{x}_r, \check{y}_r)$ can be written as:

$$\check{x}_r = \frac{1}{\|h\|} [\cos(-\angle h)x_r - \sin(-\angle h)y_r] \quad (8)$$

$$\check{y}_r = \frac{1}{\|h\|} [\sin(-\angle h)x_r + \cos(-\angle h)y_r] \quad (9)$$

The distance between any point (x_p, y_p) , belongs to line r_p , and the received symbol $(\check{x}_r, \check{y}_r)$ is:

$$d_{r,p} = \sqrt{(\check{x}_r - x_p)^2 + (\check{y}_r - y_p)^2}. \quad (10)$$

Substituting (7) in (10):

$$d_{r,p} = \sqrt{\left(\check{x}_r - \frac{-[(\log_2 M - 1) + 2(p - 1)]}{2} A\right)^2 + (\check{y}_r - y_p)^2}, \quad (11)$$

and replacing (8) and (9) in (11) and squaring both sides results:

$$\begin{aligned}
 d_{r,p}^2 = & \left(\frac{1}{\|h\|} [\cos(-\angle h)x_r - \sin(-\angle h)y_r] - \right. \\
 & \left. - \frac{[(\log_2 M - 1) + 2(p - 1)]A}{2} \right)^2 + \\
 & + \left[\frac{1}{\|h\|} [\sin(-\angle h)x_r + \cos(-\angle h)y_r] - y_p \right]^2. \tag{12}
 \end{aligned}$$

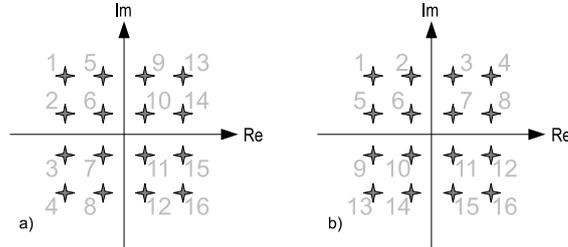
Considering that M , h and A are constants during a symbol period, we readily obtain:

$$d_{r,p}^2 = C(p) + (K - y_p)^2, \quad \text{where} \tag{13}$$

$$K = \frac{1}{\|h\|} [\sin(-\angle h)x_r + \cos(-\angle h)y_r] \tag{14}$$

$$\begin{aligned}
 C(p) = & \left(\frac{1}{\|h\|} [\cos(-\angle h)x_r - \sin(-\angle h)y_r] - \right. \\
 & \left. - \frac{[(\log_2 M - 1) + 2(p - 1)]A}{2} \right)^2 \tag{15}
 \end{aligned}$$

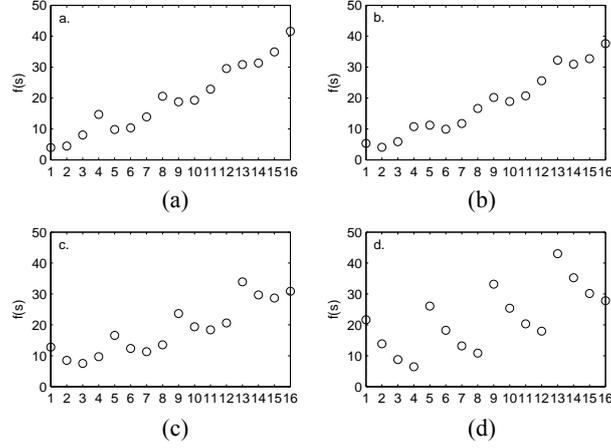
Figure 3 Two-square 16-QAM constellation mapping



Analysing equation (15) we can see that it varies only in p and K is constant during a symbol period as we can see in (14). From (13), the distance $d_{r,p}$ is $(K - y_p)^2$ summed to $C(p)$, that is constant given p fixed. Furthermore, observing Figure 3(b), the imaginary part of points $\{1, 2, 3, 4\}$ are the same and this is true for the points $\{5, 6, 7, 8\}$, and so on. The only difference between the distance from the received point to any line r_p is the constant $C(p)$ pointed out in Figure 4(b). The component $(K - y_p)^2$ is the repeating pattern that appears in Figure 4.

So, repeating patterns will appear in the evaluation of the distance between received symbol and a square M-QAM constellation with equiprobable symbols. As a result, these pattern repeats itself at each \sqrt{M} symbols.

Figure 4 Patterns in cost function depending on the distance between symbol received and distorted constellation for 16-QAM SISO system



Notes: Received symbol closer to (a) constellation point 1, (b) constellation point 2, (c) constellation point 3, (d) constellation point 4.

3.1 Extension to MIMO systems

The optimum detector that minimises the average probability of error is the ML detector. It performs the following optimisation:

$$\min_{\mathbf{s} \in \mathcal{S}} \|\mathbf{r} - \mathbf{H}\mathbf{s}\|^2 \quad (16)$$

where \mathcal{S} is the set of all possible combinations of $(s_1, s_2, \dots, s_{n_T})$ of a constellation M -QAM. Rewriting (16) we have that:

$$\min_{\mathbf{s} \in \mathcal{S}} \|\mathbf{r} - \mathbf{H}\mathbf{s}\|^2 \quad (17)$$

$$\min_{\mathbf{s} \in \mathcal{S}} \sum_{k=1}^{n_R} (r_k - \mathbf{h}_k \mathbf{s})^2 \quad (18)$$

$$\min_{\mathbf{s} \in \mathcal{S}} \sum_{k=1}^{n_R} \left[r_k - \sum_{j=1}^{n_T} h_{kj} s_j \right]^2 \quad (19)$$

$$\min_{\mathbf{s} \in \mathcal{S}} \sum_{k=1}^{n_R} [r_k - (h_{k1}s_1 + h_{k2}s_2 + \dots + h_{kn_T}s_{n_T})]^2 \quad (20)$$

Considering equation (20) and that $n_R = 1$, if we let s_1 varying and hold s_2, \dots, s_{n_T} fixed; hence, equation (20) describes the same problem of previous section (SISO system) i.e., the

distance from a point to a set of lines. Then, we can conclude that patterns will appear at each \sqrt{M} symbols. The same conclusion is valid if we vary s_2 and hold the other values variable fixed. So, to the MISO case there will be n_T -dimensional pattern on each hypercube of \sqrt{M} symbols. In the MIMO case, where $n_R > 1$, the total cost function is the sum of many cost functions that have patterns and it will also show patterns.

4 Reduced cluster search ML decoding (RCS-ML)

Our idea of cluster search arises after observing a pattern over cost functions behaviour, equations (5) and (6), considering all possible symbol combinations. As we can see from Figure 5(a), there are patterns that repeat itself. This figure was generated considering 16-QAM modulation; so 4×4 clusters can be identified.

Figure 5 Typical f_{14} values, considering (s_1, s_4) symbol pair in a 16-QAM modulation (a) for all (s_1, s_4) values, (b) for only one cluster showing local minimum, (c) for all local minima, showing global minimum

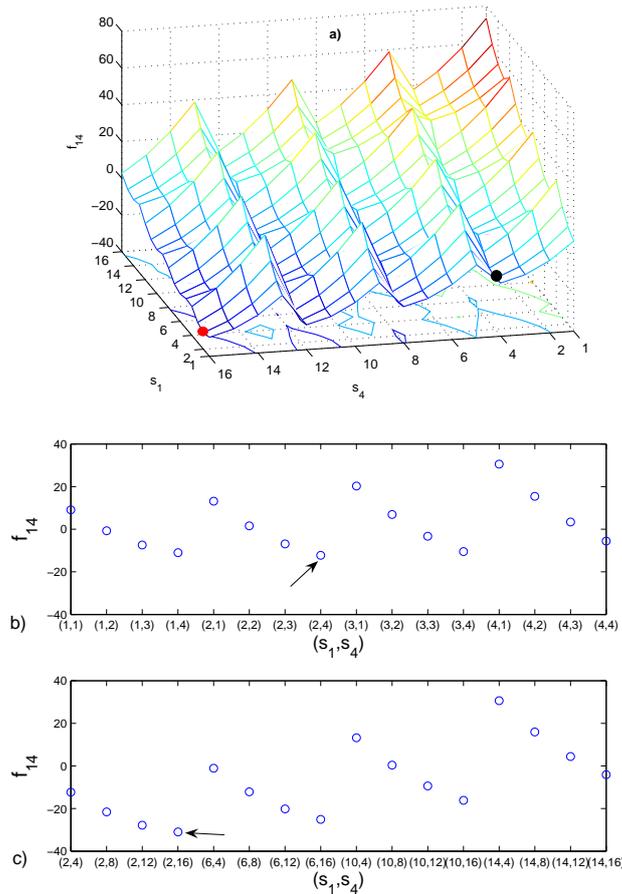
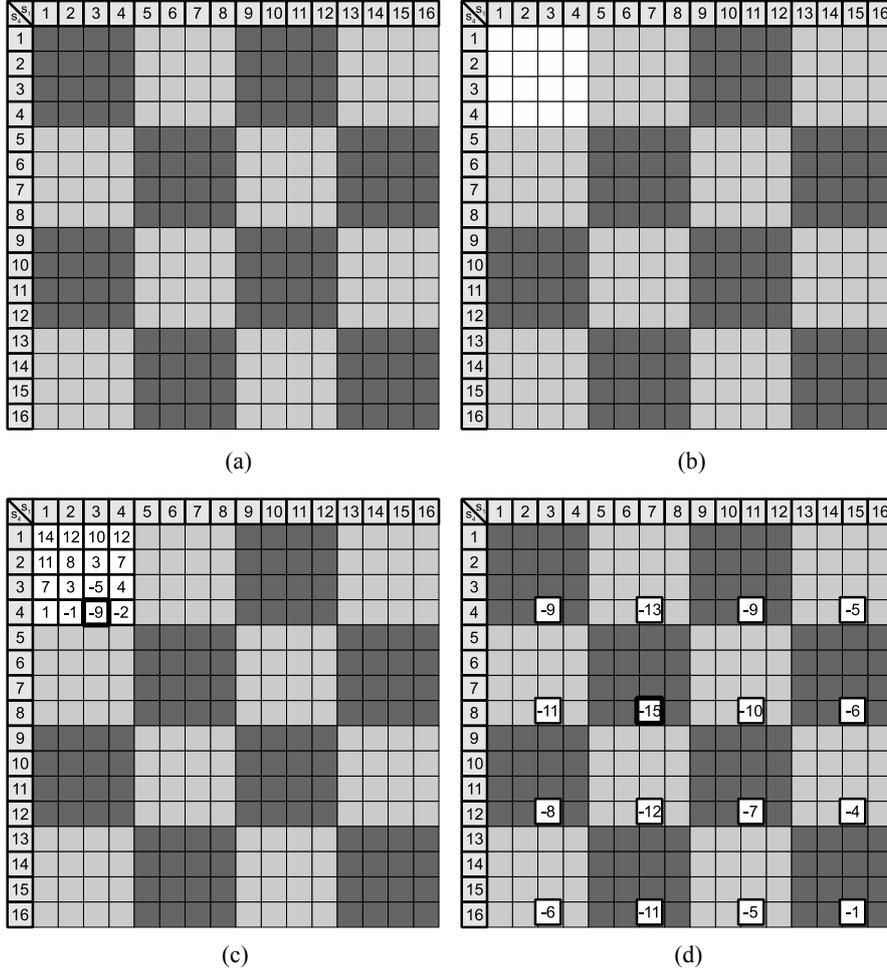


Figure 6 Example of RCS decoding and 16-QAM, (a) search space divided in clusters (b) first cluster to be explored (c) cost for each pair (s_1, s_4) inside first cluster (d) local minima with respective cost value



The idea is to perform an ML search only inside a cluster (we choose the first cluster in Figure 5(b), defining the relative coordinates of the pair that minimised (5) [or alternatively (6)] and generate a set with all pairs of symbols with the same coordinates inside the other clusters. This pattern repeats itself at each $\sqrt{2^m}$ symbols for symbol pairs (s_1, s_4) [alternatively (s_2, s_3)], and all pairs inside this pattern constitutes a cluster. Finally, perform a ML search over the generated set and choose the one that minimises (5) and (6), Figure 5(c) shows the values for $f_{1,4}$ in the cluster which global minimum occurrence ($s_4 = 16$).

In order to implement the RCS-ML QO-STBC decoding, three-steps are carry out as described in Algorithm 1. These steps are valid to find ML solution for each symbol pair, (s_1, s_4) or (s_2, s_3) in the QO-STBC scheme.

Algorithm 1 RCS-ML over (s_1, s_4) symbol pair

Input: $\mathbf{y}, \hat{\mathbf{h}}$; **Output:** (\hat{s}_1, \hat{s}_4)

- **Step 1:** perform a ML search over first cluster using equation (5). where the symbols of the M -QAM constellation are hypothetically mapped as $s_1, s_4 \in \{1, 2, \dots, \sqrt{2^m}\}$.
- **Step 2:** record the two-pairs $(\tilde{s}_1, \tilde{s}_4)$ that locally minimise equation (5).
- **Step 3:** generate sets:
 $S_i^{\text{clst}} = \{\tilde{s}_i + k\sqrt{2^m} \mid k = 0, 1, \dots, \sqrt{2^m} - 1\}, \quad i = 1, 4.$

Perform ML decoding through all possible pairs (s_1, s_4) using eq. (5), where $s_1 \in S_1^{\text{clst}}$ and $s_4 \in S_4^{\text{clst}}$.

Figure 6 shows an example of RCS-ML procedure for 16-QAM modulation. In this case, the modulation order is $m = 4$, and as $\sqrt{2^m} = 4$, then, there are 4×4 clusters, Figure 6(a). We can see from Figure 6(b). the first cluster in white color, which is composed by 16 pairs (s_1, s_4) , where $s_1, s_4 \in \{1, 2, 3, 4\}$. In Figure 6(c), a local minimum is found minimising (5), and in Figure 6(d). the global minimum is found from all clustered pairs selected in Step 3. In this case, the algorithm finds $(s_1, s_4) = (7, 8)$. This process is repeated considering the other symbol pair (s_2, s_3) and equation (6).

5 Numerical results

The main system and channel parameters used in Monte Carlo simulations are summarised in Table 1. In all numerical results shown here, QO-STBC scheme from Jafarkhani (2001) were adopted. For simplicity, in this section, only perfect know of CSI at the receiver side was assumed.

In accordance with the most common channel model in the literature (Jafarkhani, 2001; Alamouti, 1998), herein, we consider quasi-static time-varying channel model: the fading coefficients remain fixed during each QO-STBC block of $L = 4$ time slots and vary independently from one-block to the next (quasi-static fading).

Table 1 MIMO system, RCS-ML decoding and channel parameters

Parameter	Adopted values
<i>QO-STBC MIMO system</i>	
# Tx antennas	$n_T = 4$
# Rx antennas	$n_R = 1$ and 4
Modulation format	Squared M -QAM: $M = 4, 16, 64, 256$
QO-STBC code	Rate 1, $R_{\text{stbc}} = 1$ (Jafarkhani, 2001)
Received SNR per antenna	$SNR [-2.5; 30]$ dB
Throughput	$\Theta = 1.0, 2.0, 3.0, 4.0$ [bits/symb. period]
<i>Rayleigh channel</i>	
Sub-channel fading	Flat-frequency
Channel type	Quasi-static (slow), $L = 4$
Channel state info.	Perfectly known at Rx
<i>RCS-ML decoding</i>	
Cluster size	$\sqrt{2^m} \times \sqrt{2^m}$

Figure 7 describes the Monte-Carlo simulation results for the case of QO-STBC MIMO system with $n_T = 4 \times n_R = 4$ antennas employing the ML against RCS-ML decoding; while Figure 8 compares the performance of both decoders but considering $n_R = 1$ receive

antenna. One can observe that the system performances with RCS-ML and conventional ML decoding are undistinguished for all SNR levels and constellation size conditions.

Figure 7 ML against RCS-ML decoding performance comparison for the QO-STBC MIMO system; $n_T = 4 \times n_R = 4$ antennas

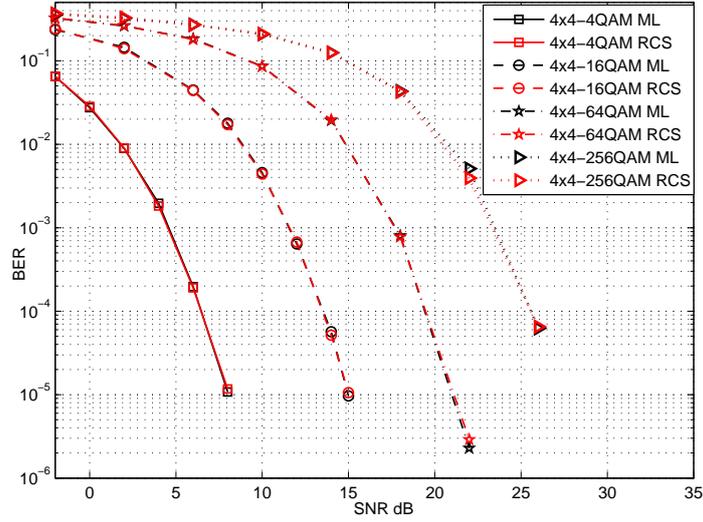
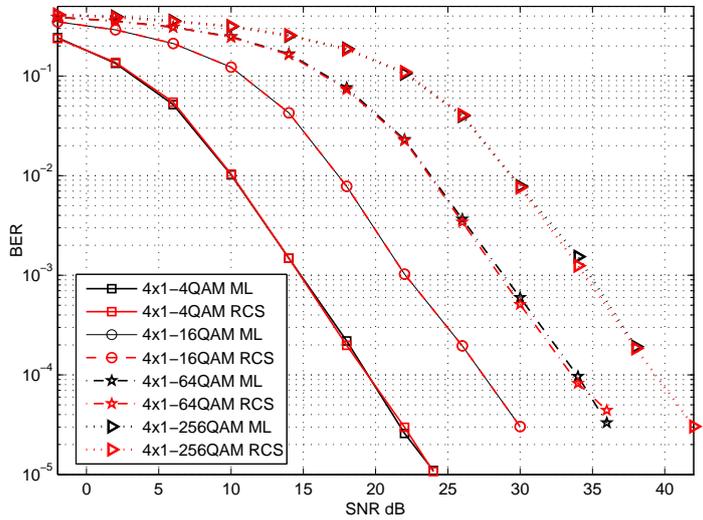


Figure 8 ML against RCS-ML decoding performance comparison for the QO-STBC MIMO system with different QAM constellation size, $n_T = 4 \times n_R = 1$ antennas



5.1 Complexity analysis for the RCS-ML decoding

In order to evaluate the algorithm complexity, we consider real multiplications and sums. Analysing (5) and (6) and taking into account each complex multiplication as four-real

multiplications and each complex sum as two-real sums, we have 90 real multiplications and 27 real sums for each f_{14} or f_{23} evaluation. Once the RCS-ML algorithm needs only 2^m ML evaluation for Step 1 and 2^m for Step 2, so, the overall ML evaluation number is only 2^{m+1} evaluation to proceed the ML QO-STBC symbol decoding. Table 2 compares our RCS-ML QO-STBC decoding complexity with that necessary to proceed the ML QO-STBC decoding. Those complexities, in terms of real multiplications and sums, that are necessary to perform QO-STBC decoding, can be compared through the perceptual complexity reduction factor, expressed by

$$CR = \frac{C_{RCS}}{C_{ML}} \times 100 \quad [\%].$$

For all the analysed constellations and number of received antennas, this complexity reduction ratio is simply given by $CR = 2^{1-m} \times 100$ [%].

Table 2 Number of real multiplications/sums per receive antenna per symbol pair necessary for QO-STBC decoding, $n_T = 4 \times n_R = 1$

Decoder	4-QAM	16-QAM	64-QAM	256-QAM
RCS	720/216	2880/864	11520/3456	46080/13824
ML	1440/432	23040/6912	368640/110592	5898240/1769472
CR_{RCS}	50%	12.5%	3.125%	0.781%

6 Conclusions

In this contribution, we have proposed a reduced complexity ML decoding scheme based on cluster search, suitable for QO-STBC MIMO systems with higher-order modulation indexes and any number of receive antennas.

Thanks to reduced cluster search procedure, the numerical results for the proposed ML decoding have indicated no degradation in the performance, achieving the ML performance in all cases analysed, with a significative increasing reduction in the computational complexity when the modulation order increases, been 12.5% of ML decoding complexity for 16-QAM, and < 1% for 256-QAM.

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