

SUB-CARRIER ALLOCATION AND PACKET SCHEDULING IN OFDMA-BASED CELLULAR NETWORKS

Emanuel B. Rodrigues, Marco Moretti, Pawel Sroka, Giulio Dainelli, and Fernando Casadevall

Departament de Teoria del
Senyal i Comunicacions (TSC)
Universitat Politècnica de
Catalunya (UPC)
emanuel@tsc.upc.edu
ferranc@tsc.upc.edu

Dipartimento di Ingegneria
dell'Informazione
Università di Pisa
marco.moretti@iet.unipi.it
giulio.dainelli@iet.unipi.it

Chair of Wireless
Communications
Poznan University of
Technology
psroka@et.put.poznan.pl

ABSTRACT

In this paper a study on different Radio Resource Allocation (RRA) methods employed in multi-carrier cellular system is presented. This work focuses on the rate-adaptive sub-carrier allocation, as well as the utility-based packet scheduling algorithms. The objective of the paper is to study the trade-off between system spectral efficiency and fairness among the users when the considered algorithms are used.

1. INTRODUCTION

The wireless shared channel in cellular networks is a medium over which many Mobile Terminals (MTs) compete for resources. In such a scenario, spectral efficiency and fairness are crucial aspects for resource allocation. The time-varying nature of the wireless environment, coupled with different channel conditions for different MTs, poses significant challenges to accomplishing these goals. In general, these objectives cannot be achieved simultaneously and an efficient trade-off must be achieved. In recent years RRA has been envisaged as one of the most efficient techniques to achieve a desirable trade-off among these three conflicting objectives in single-cell multi-carrier systems.

Many next generation wireless systems are based on Orthogonal Frequency Division Multiple Access (OFDMA), which provides a high degree of flexibility that can be exploited by RRA algorithms. There are different sources of diversity in an OFDMA-based system, such as time, frequency and multi-user diversities. Following the path open by the seminal article by Wong et al. [2], many RRA algorithms have been proposed to take advantage of these kinds of diversity. In OFDMA-based systems, it is possible to dynamically allocate subsets of sub-carriers for different MTs (Dynamic Sub-carrier Assignment (DSA)), and to adapt the Modulation and Coding Scheme (MCS) and power for each sub-carrier according to the instantaneous channel conditions (bit and power loading). Furthermore, Packet Scheduling (PSC) algorithms are responsible for deciding when the MTs will access the shared channel and with which transport format depending on the Channel State Information (CSI).

Many separate or joint solutions for the problems of DSA, bit loading and power loading were based on combinatorial optimization. Most of the works in literature follow

either the *margin adaptive* approach, formulating dynamic resource allocation with the goal of minimizing the transmitted power with a rate constraint for each user [3], or the *rate adaptive* approach aiming at maximizing the overall rate with a power constraint [4], [5]. In this latter case, the optimal solution for resource allocation in the downlink is often found as an application of the well-known waterfilling algorithm.

On the other hand, many works have been using Utility Theory to propose solutions for all the aforementioned RRA algorithms, including also multi-carrier PSC. Utility theory performs the optimization of a utility-pricing system, which is established based on the mapping of some performance criteria (e.g. rate, delay) or resource usage (e.g. sub-carriers, power) into the corresponding pricing values [1].

In this work, we will focus on the provision of Non-Real Time (NRT) services, such as World Wide Web (WWW) browsing, File Transfer Protocol (FTP) and e-mail. For these kind of services, the data rate is the most important Quality of Service (QoS) metric. The optimization problem can be formulated based on instantaneous or average data rates. The former case is stricter because QoS and fairness has to be guaranteed in each Transmission Time Interval (TTI), while the time window considered in the optimization problem based on average data rates adds a time diversity that relax the requirements on QoS and fairness.

The present work will be divided in two parts. In the first part, we will study rate adaptive sub-carrier allocation using optimization based on instantaneous data rates. In the second part, we will study multi-carrier packet scheduling using utility functions based on average data rates. The objective of the paper is to study the trade-off between system spectral efficiency and fairness among the users when the RRA algorithms mentioned above are used.

The paper is organized as follows. In section 2 the system model is described. Sections 3.1 and 3.2 present the mathematical formulation of the rate adaptive sub-carrier allocation based on instantaneous data rate and the packet scheduling based on utility theory and average data rates, respectively. The simulation results are depicted in section 4, while the conclusions are drawn in section 5.

2. SYSTEM MODEL

The considered scenario is a single cell with hexagonal shape. We consider a network with one transmitter (base-station) and J receivers (users). The transmitted Orthogonal Frequency Division Multiplexing (OFDM) signal is time-slotted, where in every time slot at most one user can be

This work was supported by the European Commission in the framework of the FP7 Network of Excellence in Wireless COMMunications NEWCOM++ (contract n. 216715).

Emanuel B. Rodrigues has a Ph.D. scholarship support by the Improvement Co-ordination of Superior Level People (CAPES) - Brazil.

served over each sub-carrier.

The considered environment is Typical Urban (TU) where each user experiences independent transmit conditions. The channel is a frequency-selective Rayleigh fading channel, with the channel coherence time such that each sub-carrier experiences only flat fading. It is assumed that the channel fading rate is slow enough so that the frequency response does not change during a TTI interval. Each user also experiences shadowing with log-normal standard deviation of 8 dB. A perfect knowledge of the CSI at the transmitter side is assumed, with no signalling overhead transmitted. The signal strength at the receiver side depends on the path-loss calculated by $L = 128.1 + 37.6 \log_{10} d$, where d is the distance to the base station in km.

The bit allocation on each sub-carrier is determined using the Shannon's capacity model shown in [1]:

$$c_{j,k} = \log_2(1 + \Gamma p_k \rho_{j,k}) \quad (1)$$

where $c_{j,k}$ is the achievable throughput of user j over sub-carrier k , p_k is the transmit power allocated at sub-carrier k , $\rho_{j,k}$ is the Signal-to-Noise Ratio (SNR) of user j at sub-carrier k , and Γ is the SNR gap given by $\frac{1.5}{-\ln 5BER}$ [1] (the considered target Bit Error Rate (BER) was 10^{-6}).

It was assumed that the MTs remained stationary, hence there is no need to implement any handover scheme. All users are assumed to have an infinite amount of data to transmit during whole simulation run (full-buffer model).

3. RESOURCE ALLOCATION ALGORITHMS

3.1 Rate adaptive sub-carrier allocation based on instantaneous data rates

RRA often leads to algorithms whose implementation is very complex. In fact the allocation problem is in general not convex since the allocation variable is integer and can assume only two values: 1 when the channel is allocated to a specific user and 0 otherwise. In most cases the optimal solution can be found only evaluating all possible allocations and the complexity grows exponentially in the number of users and sub-carriers. Therefore, most of the literature has been focused on the development of sub-optimal heuristics that have a lower computational complexity but that still yield good results. Many algorithms make the problem convex by relaxing the integer constraint on the allocation variable. Unfortunately, non-integer solutions are hardly applicable in many scenarios where a sub-carrier should be actually allocated or not to a user. In the following we will focus on the RRA problem outlining its most common formulations and solutions.

3.1.1 Sum Rate Maximization

The most common mathematical formulation of the RRA problem is

$$\begin{aligned} \max_{\mathbf{p}, \mathbf{x}} \sum_j \sum_k c_{j,k} \cdot x_{j,k} \\ \text{s.t.} \\ \sum_j x_{j,k} \leq 1 \quad \forall k \\ \sum_j \sum_k p_{j,k} \leq P_{\max} \\ x_{j,k} \in \{0, 1\} \quad \forall j, k \end{aligned} \quad (2)$$

where P_{\max} is the maximum allowed transmit power of the Base Station (BS). The optimization variables are \mathbf{x} , the vector of the allocations, and \mathbf{p} , the vector containing the power levels of all sub-carriers.

In its original formulation the problem (2) has been solved in [4] by assigning each sub-carrier to the user that maximizes its gain on it and then performing waterfilling over all the sub-carriers. On one hand, such a solution maximizes the cell throughput but on the other hand is extremely unfair tending to privilege the users that are closest to the BS and neglecting all the others.

3.1.2 Max-Min Rate Adaptive

The RRA allocation (2) tends to starve the users with the worse channel gains, i.e. the users that are more distant from the BS. Thus, in [5] the RRA problem has been formulated with the goal of maximizing the minimum capacity offered to each user, thus introducing fairness among the users. In general, fairness among the MTs comes at the cost of a decreased overall throughput of the cell. The max-min RRA problem is formulated as follows

$$\begin{aligned} \max_{\mathbf{p}, \mathbf{x}} \min_j c_{j,k} \cdot x_{j,k} \\ \text{s.t.} \\ \sum_j x_{j,k} \leq 1, \quad \forall k \\ \sum_j \sum_k p_{j,k} \leq P_{\max} \\ x_{j,k} \in \{0, 1\} \quad \forall j, k \end{aligned} \quad (3)$$

Unfortunately, the problem in the formulation (3) is not convex and the authors in [5] study an heuristic that is based on: a) transmitting the same amount of power (P_{\max}/K) on each channel; b) implementing an *assignment* strategy that iteratively assigns each sub-carrier to the user with the smallest rate.

3.1.3 Sum Rate Maximization with Proportional Rate Constraints

The max-min RRA (3) guarantees that all users achieve a similar data rate. However, different users may require different data rates. In this case the max-min solution is not able to comply with the different user requirements. The RRA algorithm presented in [6] is designed to allocate radio resources proportionally to different rate constraints that reflect different levels of service. The RRA problem is formulated as follows

$$\begin{aligned} \max_{\mathbf{p}, \mathbf{x}} \sum_j \sum_k c_{j,k} \cdot x_{j,k} \\ \text{s.t.} \\ \sum_j x_{j,k} \leq 1, \quad \forall k \\ \sum_j \sum_k p_{j,k} \leq P_{\max} \\ x_{j,k} \in \{0, 1\} \quad \forall j, k \\ R_1 : R_2 : \dots : R_J = \gamma_1 : \gamma_2 : \dots : \gamma_J \end{aligned} \quad (4)$$

where R_j indicates the rate for user j , defined as $R_j = \sum_k c_{j,k} \cdot x_{j,k}$ and γ_j ($j = 1, \dots, J$) is a set of predetermined values that are used to ensure proportional fairness among users. The optimization in (4) is a mixed binary integer programming problem and as such is in general very hard to solve.

Thus, also in this case the problem is solved using a sub-optimal heuristic and the optimization (4) is performed in two steps. In the first step, following the approach taken in [5], the sub-carriers are allocated trying to comply as much as possible with the proportional rate constraints and assuming a uniform power distribution. In the second step, having fixed the sub-carrier allocation, the power is distributed to the users so that the proportional rate constraints are met exactly.

3.2 Packet Scheduling Based on Utility Theory and Average Data Rates

In this section we formulate PSC algorithms that use Utility Theory in order to find an efficient trade-off between system spectral efficiency and fairness among the users. The considered optimization problem is the maximization of the total utility with respect to the throughput (average data rate), which is calculated using a low-pass Simple Exponential Smoothing (SES) filtering as indicated in [1].

Assuming that the time constant of the exponential filter is sufficiently large, it is proven in [1] that the DSA problem has a closed form solution. The MT j^* is chosen to transmit on the k th sub-carrier in TTI n if it satisfies the condition given by (5):

$$j^* = \arg \max_j \left\{ U_j'(T_j[n-1]) \cdot c_{j,k}[n] \right\}, \quad \forall j \quad (5)$$

where $U_j'(\cdot)$ is the marginal utility of the j th MT, $T_j[n-1]$ is the throughput of the j th MT up to TTI $n-1$, and $c_{j,k}[n]$ denotes the instantaneous achievable transmission efficiency of the j th MT on the k th sub-carrier.

We will consider a family of utility functions of the form presented in (6) below [9].

$$U_j(T_j[n]) = \frac{T_j[n]^{1-\alpha}}{1-\alpha} \quad (6)$$

where α is a non-negative parameter that determines the degree of fairness. The fairness of the utility function becomes stricter as α increases.

According to (5), this is equivalent to consider a priority function of the PSC algorithm given by:

$$P_{j,k}^{PSC} = \frac{c_{j,k}[n]}{T_j[n-1]^\alpha}, \quad \forall j, k; \quad \alpha \in [0, \infty) \quad (7)$$

For each of the K sub-carriers in the system, a multi-carrier PSC algorithm calculates the priority functions for all J MTs according to (7) and assign it to the MT that has the highest priority value.

We will show in sections 3.2.1, 3.2.2 and 3.2.3 that, depending on the value of the parameter α , the general utility framework presented above can be designed to work as any of three well-known classical PSC algorithms: Max-Rate (MR), Max-Min Fairness (MMF) and Proportional Fairness (PF). Furthermore, in section 3.2.4 we present the Adaptive Fairness (AF) PSC algorithm, which can achieve an adaptive trade-off between spectral efficiency and fairness according to the cellular operator's objectives.

3.2.1 Rate Maximization

The MR PSC algorithm is able to maximize the system spectral efficiency because it considers a linear utility function

$U_j(T_j[n]) = T_j[n]$, which yields a constant marginal utility $U_j'(T_j[n]) = 1$ [1]. One can notice that this can be achieved setting $\alpha = 0$ in (6). According to (7), this is equivalent to consider a priority function related to the MR algorithm given by (8) below.

$$P_{j,k}^{MR} = c_{j,k}[n], \quad \forall j, k \quad (8)$$

As the final result, each sub-carrier will be assigned to the MT that has the highest channel gain on it. The MR criterion maximizes the system capacity at the cost of unfairness among the MTs, because those with poor radio link quality will probably not have chance to transmit.

3.2.2 Max-Min Fairness

The utility function of the MMF algorithm is the limit of the function in (6), when $\alpha \rightarrow \infty$ [9]. According to (5) and (7), the priority function is dependent on the marginal utility $U_j'(T_j[n])$ and the achievable instantaneous transmission efficiency $c_{j,k}[n]$. However, in the case of the MMF criteria and when considering MTs with lower data rates, the influence of the marginal utility when $\alpha \rightarrow \infty$ is so high that the influence of the channel quality becomes negligible. Taking this fact into account, we can assume a more simplified priority function for the MMF algorithm given in (9), which is also known in the literature as the "Fair Throughput" criterion [7].

$$P_{j,k}^{MMF} = \frac{1}{T_j[n-1]}, \quad \forall j, k \quad (9)$$

which gives priority to the MT that has experienced the worst throughput so far. In this way, in terms of throughput, it is the most fair criterion possible, since all MTs will have approximately the same throughput in the long-term. However, since this criterion maximizes the throughput of the worst MTs, it will provide low aggregate system throughput.

3.2.3 Proportional Fairness

A trade-off between spectral efficiency and fairness can be achieved by means of a PF PSC algorithm [8]. In utility theory, the logarithmic utility function is associated with the proportional fairness [1]. In the general family of utility functions presented in (6), the logarithmic function can be achieved when $\alpha \rightarrow 1$ (see proof on [9]). Therefore, according to (7), the priority function of the PF algorithm is given by (10).

$$P_{j,k}^{PF} = \frac{c_{j,k}[n]}{T_j[n-1]}, \quad \forall j, k \quad (10)$$

3.2.4 Adaptive Fairness

The AF PSC algorithm, which was proposed in [10], joins in a unified framework the three aforementioned classical PSC algorithms (MR, MMF and PF). In the light of utility theory, it was shown that a general PSC algorithm based on (6) is able to provide several degrees of fairness. The AF algorithm adaptively explores this flexibility in order to achieve an efficient trade-off between spectral efficiency and fairness planned by the network operator. However, it is difficult to design an adaptive control of the α parameter because it is defined over a large range of values. Instead of that, the AF algorithm transforms the priority function of (7) into another

priority function that is based on a parameter β , which is defined over a controlled range and provides the possibility of a stable and simple adaptive control. The priority function of the AF algorithm is presented in (11) below.

$$P_{j,k}^{AF} = \frac{c_{j,k}[n]^{1-\beta}}{T_j[n-1]^\beta}, \quad \forall j,k; \quad \beta \in [0,1] \quad (11)$$

Notice that in a conceptual point of view, the priority functions in (7) and (11) perform in the same way. The AF algorithm is able to work as the classical PSC algorithms by means of the adaptation of the β parameter. The values of $\beta = \{0, 0.5, 1\}$ corresponds to the MR, PF and MMF, respectively. For more details on how the β parameter is adapted in a feedback control loop, see [10].

4. SIMULATION RESULTS

In this section the simulation-specific parameters as well as the simulation results are presented. The simulation results for the sub-carrier allocation and packet scheduling algorithms are depicted in sections 4.1 and 4.2, respectively.

The cell radius was set to 500 m, and there were 192 sub-carriers available for data transmission. The data sub-carriers spacing was set to 15 kHz, giving the total transmit bandwidth of 2.88 MHz, with the centre frequency of 2 GHz. The TTI interval was equal to $T_s=0.5$ ms, with 20 consecutive time slots constituting one superframe, and each of the TTIs consisting of 7 OFDM symbols [11]. The Additive White Gaussian Noise (AWGN) power on each sub-carrier was assumed to be equal to -123.24 dBm.

4.1 Rate Adaptive Sub-carrier Allocation

The results presented in this section are obtained for all RRA algorithms presented in Section 3.1 considering 1 W transmit power and averaged over 500 realizations. For the sum rate maximization with proportional rate constraints two different sets of rate constraints have been studied: the case where the rate constraints are set equal for all users (Prop rate 1), i.e. $\gamma_j = 1$ ($j = 1, \dots, J$), and the case where the rate constraints are set proportional to the user pathloss (Prop rate 2).

Fig. 1 shows the average throughput for the different algorithms: the sum rate maximization algorithm achieves the highest throughput and the max-min the lowest. The results show also the flexibility of the algorithm with proportional rate constraints. As expected, when the set of rate constraints are all equal its behavior is almost identical to the max-min algorithm. On the other hand, when the system tends to favor the users nearer to the BS, the throughput approaches the sum rate results.

Fig. 2 shows the average fairness index (according to the definition in [12]) for the various RRA algorithms. In this case the max-min and the algorithm with equal rate constraints outperform all the others. The algorithm with rate constraints proportional to the pathloss even if guarantees access to all users is not very fair. This is due to the fact that in our simulation setting the difference in pathloss can be several orders of magnitude large. Thus, users close to cell boundaries will have a much smaller throughput than users near the BS.

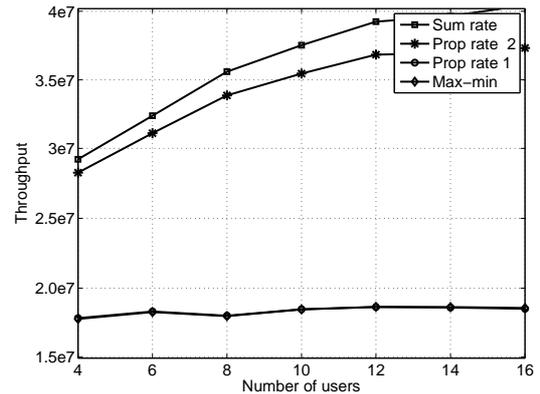


Figure 1: Achievable throughput as function of the number of users

4.2 Utility-Based Packet Scheduling

In case of the utility-based packet scheduling analysis the power distribution over all sub-carriers was uniform with no power adaptation. Only 3 modulation schemes have been taken into account: Quadrature Phase Shift Keying (QPSK), 16-Quadrature Amplitude Modulation (QAM) and 64-QAM. The transmitter selects one of the three on the base of the ideally achievable throughput (1) based on SNR. For each simulation point 10 different realizations have been considered, with the simulation time span for each of the realizations set to 30 s (60000 TTIs).

Fig. 3 shows the system fairness index described in [12] for different cell loads. In this simulation scenario, the AF target fairness index is 0.9. It can be observed that AF is successful to achieve its main objective, which is to guarantee a strict fairness distribution among the MTs. This is achieved due to the feedback control loop that dynamically adapts the parameter β of the AF priority function (11). As expected, MMF provided the highest fairness, very close to the maximum value of 1, while MR proved to be the most unfair strategy with a high variance on the fairness distribution for high cell loads. PF presented a good fairness distribution, close

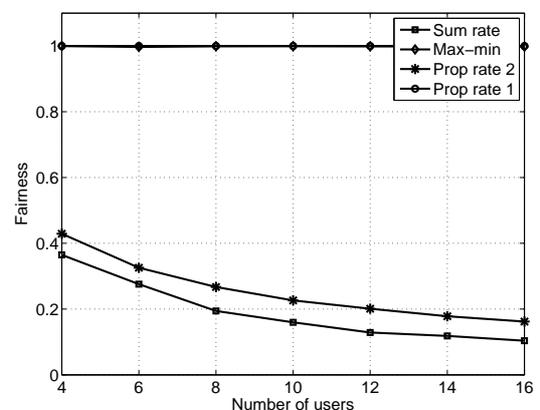


Figure 2: Measured fairness as function of the number of users

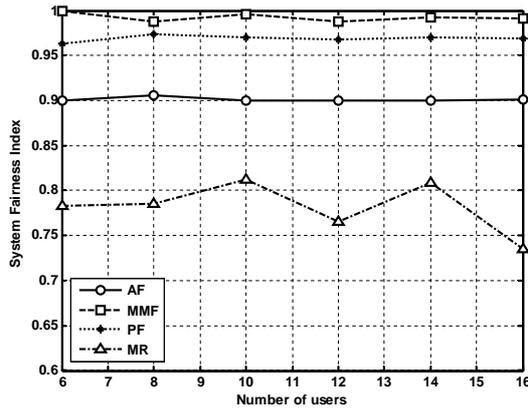


Figure 3: Comparison of utility-based packet scheduling algorithms regarding the system fairness index

to the performance of MMF. The advantage of the AF algorithm in comparison with the others is that it can be designed to provide any required fairness distribution, while the classical PSC strategies are static and do not have the freedom to adapt themselves and guarantee a specific performance result.

The total cell throughput for different cell loads is shown in Fig. 4. As expected, MR was able to maximize the spectral efficiency, while MMF presented the lowest cell throughput. Since PF is a trade-off between MR and MMF, its performance lied between them. Looking at Fig. 3, one can expect that since the AF target fairness index was set to 0.9, the AF performance would be somewhere in the middle between the performances of PF and MR. This can be observed in Fig. 4, where the total cell throughput is plotted. Notice that the throughput values presented in this section are lower than those shown in section 4.1. This was due to the fact that the simulations carried out for the PSC investigation considered three discrete modulations while the simulations of section 4.1 assumed continuous link adaptation with no rate restrictions.

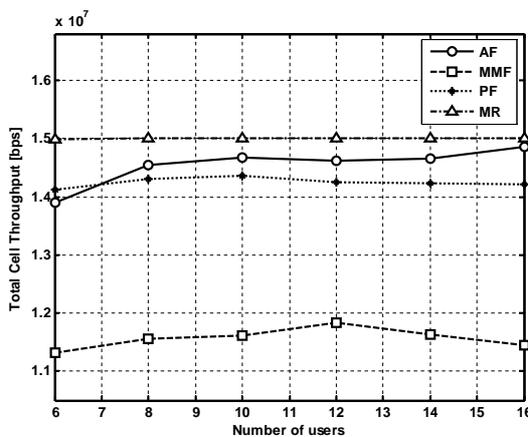


Figure 4: Comparison of utility-based packet scheduling algorithms regarding the total cell throughput

5. CONCLUSIONS

In this paper we investigated the trade-off between system spectral efficiency and fairness among users in OFDMA-based cellular networks. It was concluded from simulation results in a single-cell scenario that it is possible to achieve an efficient trade-off using rate adaptive sub-carrier allocation based on instantaneous data rate or utility-based packet scheduling based on average data rate (throughput).

REFERENCES

- [1] G. Song and Y. G. Li, "Cross-layer optimization for OFDM wireless networks - part I: Theoretical framework and part II: Algorithm development," *IEEE Transactions on Wireless Communications*, vol. 4, no. 2, pp. 614–634, Mar. 2005.
- [2] C.Y. Wong, R.S. Cheng, K.B. Letaief and R.D. Murch, "Multiuser OFDM with Adaptive Subcarrier, Bit and Power Allocation," *IEEE J. Select. Areas Commun.*, vol. 17, no. 10, pp. 1747-1758, Oct. 1999.
- [3] M. Moretti and M. Morelli, "A novel dynamic sub-carrier assignment scheme for multiuser OFDMA systems," *Proc. VTC 2006-Spring*, May 2006.
- [4] J. Jang and K. B. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 2, pp. 171–178, 2003.
- [5] W. Rhee and J. M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," in *Proc. IEEE 51st Vehicular Technology Conference - VTC Spring*, 2000, pp. 1085–1089.
- [6] Z. Shen and J. G. Andrews and B. L. Evans, "Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints," *IEEE Transactions on Wireless Communications*, vol. 4, no. 6, pp. 2726–2737, 2005.
- [7] P. J. G. Ameigeiras, *Packet Scheduling and Quality of Service in HSDPA*, Ph.D. dissertation, Aalborg University, Aalborg, Denmark, 2003.
- [8] F. Kelly, "Charging and Rate Control for Elastic Traffic," *European Transactions on Communications*, vol. 8, pp. 33–37, 1997.
- [9] S. Shakkottai and R. Srikant, "Network Optimization and Control," *Foundations and Trends in Networking*, vol. 2, no. 3, pp. 271–379, 2007.
- [10] E. B. Rodrigues and F. Casadevall, "Adaptive Radio Resource Allocation Framework for Multi-User OFDM," accepted for publication in *IEEE 69th Vehicular Technology Conference - VTC Spring*, Barcelona, Spain, April, 2009.
- [11] 3GPP, "Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation," TS 36.211 v8.4.0, September, 2008.
- [12] J. Tang and X. Zhang, "Cross-Layer Design of Dynamic Resource Allocation with Diverse QoS Guarantees for MIMO-OFDM Wireless Networks," in *Proc. Sixth IEEE International Symposium on a World of Wireless Mobile and Multimedia Networks - WoWMoM*, 2005, pp. 205–212.