Transmitter Power Control for DS/CDMA Cellular Mobile Radio Networks

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Abstract - **For a given channel allocation, the capacity and quality of communications of cellular radio systems using Direct Sequence/Code-Division Multiple-Access (DS/CDMA) can be increased by using a transmitter power control scheme to combat the near-far problem. Centralized and distributed transmitter power control schemes to minimize the outage probability due to cochannel interference in cellular radio systems have been investigated in [5]-[10]. In these papers, however, the effect of adjacent channel interference (near-far problem) is neglected. The goal of our paper is, based on the models and ideas in [5]-[10], to investigate transmitter power control strategies appropriate for DS/CDMA high capacity cellular radio systems. Monte-Carlo simulation results of these power control schemes indicate that substantial system capacity gains can be obtained with respect to conventional systems. It is also shown that the performance of properly designed distributed algorithms can closely approximate that of the optimum centralized power control scheme.**

I. INTRODUCTION

Direct-Sequence/Code-Division Multiple-Access (DS/CDMA) has been proposed for wireless mobile as well as personal communications applications [1]-[2]. DS/CDMA communications systems achieve multiple-access capability by assigning a distinct signature waveform to each user from a set of waveforms with low mutual cross correlations. Because of the lack of orthogonality between signals, performance is affected by the so-called *near-far problem*. In fact, the adjacent channel interference (ACI), or near-far problem, is the single most restraining factor on the system capacity of current mobile radio networks using DS/CDMA [3]. To alleviate the severe reduction of multiple-access capability caused by the near-far problem and its ubiquity in networks with dynamically changing topologies, transmitter power control and the design of signature waveforms with more stringent cross correlation properties have been a target of researchers in the area for several years and the only remedies implemented in practice.

The central idea in power control schemes is to maximize the minimum *carrier-to-interference ratio* (CIR) in each of the *channels* in the system. Most of the early work in this area relied on power control strategies in which transmitters were required to adaptively adjust power levels so that all signals arrive at the receiver with equal powers [4]. However, such strategies have only limited ability to increase the multiple access capability of the overall system. In [5], Aein investigates cochannel interference management in satellite networks. In his work he proposes controlling the transmitter power in order to achieve a *balanced* CIR. In [5], Nettleton and Alavi apply and extend these results to spread spectrum cellular radio systems. Using the basic ideas in [5]-[6], Zander [7] derives an optimum power control scheme which maximizes the outage probability i.e., the probability that a randomly chosen link is subject to excessive interference. However, this scheme is found to be somewhat impractical due to its computational complexity. For this reason, Zander proposes a simple sequential procedure, called the *stepwise removal algorithm (SRA)*, for practical implementations. This procedure is slightly improved by the *stepwise maximum-interference removal algorithm (SMIRA)* proposed by Lin *et al.* in [8]. Based on the results in [5]-[7], Zander [9] and Grandhi *et al.* [10] investigate distributed power control algorithms that successfully approximate the behavior of the optimum centralized algorithm by using only limited path gain information. In their papers, however, Zander [7] [9], Lin *et al.* [8] and Grandhi *et al.* [10] assume a cellular system using *orthogonal* channels, thus neglecting the effect of the near-far problem. In our paper, based on the models and ideas in [5]-[10], the performance of transmitter power control algorithms in DS/CDMA high capacity cellular radio systems, where this assumption may not be very realistic, is investigated.

The organization of the rest of the paper is as follows. In Section II, we describe the system model and define the system performance measure that will be used throughout the paper. Based on some results from the theory of positive matrices, Section III is devoted to derive the *optimum* carrier-to-adjacent channel interference achievable in a DS/CDMA cellular mobile radio system. Sections IV and V deal with the development of centralized and distributed power control algorithms, respectively. Finally, in Section VI, numerical Monte-Carlo simulation results comparing the performance of these algorithms are presented, showing that the performance of the distributed algorithm closely approximates that of the optimum centralized scheme.

II. SYSTEM MODEL

Let us consider a high capacity interference limited cellular radio system consisting of N cells. Also, let us assume that for a given channel allocation scheme, L_i mobile units are assigned to base station i, $1 \leq i \leq N$, for communication. Without

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any loss of generality, we restrict our analysis to the up-link (mobile-to-base). All the results in this paper can be applied to the down-link by changing the notation [6] [11].

In DS/CDMA systems, due to lack of orthogonality between spreading sequences, all active users in the system interfere each other. Then, we may express the CIR at the receiver of the i-th cell base station corresponding to mobile k assigned to this cell as

$$\Gamma_{ik} = \frac{P_{ik}G_{iki}}{\sum_{j=1}^{N}\sum_{l=1}^{L_j} P_{jl}G_{jli} - P_{ik}G_{iki}}, \quad 1 \le k \le L_i, \; 1 \le i \le N \tag{1}$$

where P_{jl} represents the transmitted power from mobile l, $1 \le l \le L_j$, assigned to the j-th cell base station and G_{jli} represents the gain on the link between the l-th mobile of the j-th cell, and the base station of the i-th cell. Each mobile in the system will select the serving base station according to the strongest local-mean signal based on down-link measurements of relative (pilot) signal power.

Let Γ denote the CIR at some randomly chosen base station receiver. The cumulative probability distribution function (cdf) of Γ can be defined as

$$F(\gamma) = \frac{1}{Q}\sum_{i=1}^{N}\sum_{k=1}^{L_i} F_{ik}(\gamma) \tag{2}$$

where

$$Q = \sum_{i=1}^{N} L_i \tag{3}$$

and $F_{ik}(\gamma)$ is the cdf of Γ_{ik}. Now, assuming that the transmission system requires a threshold CIR (*system protection ratio*) γ_0, then the outage probability, defined as the probability that some randomly chosen base station receiver has a CIR below the system protection ratio, will be given by

$$F(\gamma_0) = Pr\{\Gamma \le \gamma_0\} \tag{4}$$

This is the performance measure that will be used throughout the paper.

III. ACHIEVABLE ADJACENT CHANNEL INTERFERENCE

A CIR γ is defined to be *achievable* if there exists a set of non-negative transmitted powers $\{P_{ik}\}$ such that $\Gamma_{ik} \ge \gamma$ for all $1 \le i \le N$ and $1 \le k \le L_i$. Zander [7] shows that the maximum achievable CIR, denoted by γ^*, is reached with a CIR *balanced system* as proposed in [5]-[6]. To balance the CIR for each mobile in a cell we set $\Gamma_{ik} = \Gamma_i$ for all $1 \le i \le N$ and $1 \le k \le L_i$. This is equivalent to assuming that the power received in a cell site coming from any mobile unit assigned to this cell is constant, that is,

$$P_{jl} = \frac{R_j}{G_{jlj}}, \quad 1 \le j \le N, \quad 1 \le l \le L_j. \tag{5}$$

Thus, γ will be achievable if the following inequalities hold,

$$\Gamma_i = \frac{R_i}{\sum_{j=1}^{N} R_j \sum_{l=1}^{L_j} \frac{G_{jli}}{G_{jlj}} - R_i} = \frac{R_i}{\sum_{j=1}^{N} R_j Z_{ij} - R_i} \ge \gamma, \quad 1 \le i \le N \tag{6}$$

When all inequalities (6) are satisfied with equality (*fully balanced system*), this can be rewritten in matrix form as

$$\frac{1+\gamma}{\gamma}\boldsymbol{R} = \boldsymbol{Z}\boldsymbol{R} \tag{7}$$

where $\boldsymbol{R} = \{R_i\}$ denotes the base station received power vector and $\boldsymbol{Z} = \{Z_{ij}\}$ is a $N \times N$ square matrix of random variables defined as the *normalized link gain matrix.* Thus, the largest achievable CIR is given by $\gamma^* = 1/(\lambda^* - 1)$, where λ^* is the largest real eigenvalue of $\boldsymbol{Z}$. The received power vector $\boldsymbol{R}^*$ achieving this maximum is the eigenvector corresponding to λ^*. The existence of exactly one real positive eigenvalue λ^*, for which the corresponding eigenvector is positive (i.e., all components have the same sign), is guaranteed by the Perron, Frobenius and Wielandt theory of positive matrices [12].

IV. CENTRALIZED POWER CONTROL ALGORITHMS

A centralized *power control algorithm* (PCA) can be defined as an algorithm that in every moment has access to the entire normalized link gain matrix $\mathbf{Z}$ and may instantaneously control the entire base station received power vector $\boldsymbol{R}$ and thus, the entire set of mobile unit transmitted powers. From (2) it is obvious that the outage probability can be optimized by minimizing the number of links in which the protection ratio γ_0 may not be achieved (*unusable* links). In fact, an *optimum* centralized PCA will, by removing as few links as possible, find a matrix $\boldsymbol{Z}^*$ for which γ_0 is achievable. When the adjacent channel interference is neglected, as it is done by Zander in [7] and Lin *et al.* in [8], $\boldsymbol{Z}^*$ is a submatrix of $\mathbf{Z}$ where all rows and columns corresponding to zero components in the optimum transmitted power vector have been removed. However, in DS/CDMA systems $\boldsymbol{Z}^*$ will be a $M \times M$ square matrix, $M \le N$ being equal to the number of cells having at least one *usable* link, whose elements are computed by removing all the components corresponding to *unusable* links from the set of mobile transmitted powers.

To implement such an optimum procedure, a *brute force algorithm* (BFA) would first check if γ_0 is achievable for the original matrix $\mathbf{Z}$. If not, it would try to remove all combinations of at most Q-2 links, computing the eigenvalue of each *reduced* system until the CIR requirement was fulfilled. Clearly, such an algorithm would find an optimum base station received vector in a finite number of steps since finally, by removing combinations of Q-1 links no interference would remain. When implementing the BFA to remove *unusable* links, however, the total number of eigenvalue computations that have to be performed in the *worst case* is equal to 2^Q-Q-2 [8], that is, it grows exponentially with Q. To overcome this practical problem the *SRA* [7] and *SMIRA* [8] sequential procedures can be modified to be used in DS/CDMA systems.

DS/CDMA-Stepwise Removal Algorithm (DS/CDMA-SRA)

Step 1: Determine γ^* corresponding to the original matrix Z. If $\gamma^* \ge \gamma_0$, then use the eigenvector $\boldsymbol{R}^*$ and stop, else set M=N and perform step 2.

Step 2: Remove the link *ik* for which the maximum of the sums

$$r_{ik}=\sum_{j=1}^{M}\sum_{l=1}^{L_j}\frac{G_{jli}}{G_{iki}}\ ,\quad r_{ik}^{T}=\sum_{j=1}^{M}\sum_{l=1}^{L_j}\frac{G_{ikj}}{G_{jlj}} \tag{8}$$

is maximized (combined sum criterion), check if M has to be changed and form the $M\times M$ matrix Z^*. Determine γ^* corresponding to Z^*. If $\gamma^*\geq\gamma_0$, then use the corresponding eigenvector R^* and stop, else repeat step 2.

By this procedure, one by one, *unusable* links are removed stepwise until all the CIRs in all remaining links are larger than the system protection ratio. As it can be seen, each step involves only one eigenvalue computation. Thus the complexity of this algorithm grows linearly with Q.

DS/CDMA-Stepwise Maximum-Interference Removal Algorithm (DS/CDMA-SMIRA)

The fact that the larger the transmitter power the greater the interference it causes on other links in the system can be exploited to improve the performance of the *DS/CDMA-SRA* algorithm. That is, if a mobile unit which uses a high transmitting power is removed, it is likely that the CIRs in all remaining links in the system can achieve the system protection ratio. The *DS/CDMA-SMIRA* algorithm is the same as the *DS/CDMA-SRA* algorithm except that in step 2 it removes the active links *ik* stepwise for which the maximum of the sums

$$r_{ik}=\sum_{j=1,j\neq i}^{M}\ \sum_{l=1,l\neq k}^{L_j}P_{jl}\frac{G_{jli}}{G_{iki}}\ ,\quad r_{ik}^{T}=P_{ik}\sum_{j=1,j\neq i}^{M}\ \sum_{l=1,l\neq k}^{L_j}\frac{G_{ikj}}{G_{jlj}} \tag{9}$$

is maximized. That is, the *DS/CDMA-SMIRA* algorithm considers transmitter power to be an important factor for the criterion of removing links, providing better outage probability with only a slight increase in computational complexity. The term 'maximum interference' [8] is used because r_{ik} represents the total interference at the receiver of the *i*-th cell base station corresponding to mobile k assigned to this cell and r^T_{ik} represents the total interference to other base station receivers caused by the *k-th* mobile in cell *i*.

V. DISTRIBUTED POWER CONTROL ALGORITHMS

The application of the previously proposed algorithms would require reliable measurements of the gains in all propagation links in the system. Therefore, these algorithms serve mainly as tools to derive upper bounds on the performance of transmitter power control algorithms rather than being suited for practical implementation. In this section, a distributed discrete time power control algorithm that approximates the behavior of the centralized DS/CDMA-SMIRA algorithm by using only limited path gain information, is presented.

In a distributed discrete time power control scheme the base stations adjust their received powers synchronously in discrete time steps. Consider the following algorithm, in which the received power adjustment made by the *i*-th base station at the υ-th instant is given by

$$R_i^{(\nu+1)}=\xi^{(\nu)}R_i^{(\nu)}\left(1+\frac{1}{\Gamma_i^{(\nu)}}\right)=\xi^{(\nu)}\sum_{j=1}^{N}R_j^{(\nu)}Z_{ij}\ ,\quad \nu\geq 1 \tag{10}$$

The algorithm starts with an arbitrary positive vector $R^{(0)}$. In matrix form equation (10) can be expressed as

$$R^{(\nu+1)}=\xi^{(\nu)}ZR^{(\nu)}\ ,\quad \nu\geq 1 \tag{11}$$

This iteration represents the *power method* for finding the largest eigenvalue and the corresponding eigenvector of the Z matrix, thus it converges to the optimal *balanced* solution and can be considered as a *distributed balancing algorithm*. The $\xi^{(\nu)}$s need to be properly chosen to prevent the transmitter powers from becoming too large or too small. A possible choice for $\xi^{(\nu)}$ could be $1/|R^{(\nu)}|$. This would ensure a *constant* average power level but at the expense of some sort of communications between the base stations.

In Section IV it has been shown that, assuming a CIR balanced system, the outage probability can be optimized by minimizing the number of links in which the protection ratio γ_0 may not be achieved. Further, simple sequential procedures of roughly linear complexity, designed to approximate the optimum power control algorithm, have been introduced. To be able to use a similar procedure in a DS/CDMA system with a distributed power control environment we now propose the following algorithm.

DS/CDMA-Distributed Balancing Stepwise Maximum Interference Removal Algorithm (DS/CDMA-DBSMIRA)

Step 1: Set $M=N$.

Step 2: Set $R=1$. Measure the CIR vector $\Gamma^{(0)}$. If $\Gamma_i^{(0)}>\gamma_0$ for all *i*, stop; otherwise perform step 3.

Step 3: Operate the distributed balancing algorithm for at most *m* steps. If at some step $\nu<m$, $\Gamma_i^{(\nu)}>\gamma_0$ for all *i*, stop; otherwise perform step 4.

Step 4: Remove the link *ik* in the cell *i* corresponding to the mobile unit *k* with the highest transmitter power in the system. Check if M has to be changed, form the matrix Z^* and go to step 2.

The base station received power vector is reset to equal power in all cells in the system after each *unusable* link removal . The removal criterion removes the link corresponding to the mobile unit with the highest transmitter power in the system. After the link removal, the distributed balancing algorithm is reapplied on the matrix Z^* whose elements are computed by removing the component corresponding to the *unusable* link from the set of mobile transmitted powers.

We have to take into account that in a mobile environment the normalized link gain matrix is constantly changing. Then, the maximum number of iterations we allow for balancing can be a critical parameter for the *DS/CDMA-DBSMIRA* procedure. In spite of that, the speed of convergence of the power method depends on how much larger the largest eigenvalue is than the other ones. Since for mobile radio networks using DS/CDMA, assuming typical performance requirements, processing gain and modulation, coding and equalization schemes, the system protection ratio will be far less than one, then all the eigenvalues of **Z** will be close to zero. In this way, this distributed power control algorithm has the potential to offer a high speed of convergence. Therefore, it should permit to work with balancing periods shorter than the coherence time of the path gain in different wireless mobile environments.

Since the removal procedure requires the comparison of the maximum transmitter power values in all the cells in the system, the *DS/CDMA-DBSMIRA* algorithm could, in a sense, be considered as a centralized power control algorithm. Nevertheless, as is stated by Zander in [7], the removal procedure is already part of the normal handoff/dynamic assignment procedures of the system. Therefore, this power control scheme should not impose any additional computational burden on the system.

VI. NUMERICAL RESULTS

Numerical results have been obtained for a cellular scenario where a *minimal nontrivial service area*; namely, nineteen identical hexagonal cells, arranged in three concentric rings, has been considered. Base stations use omnidirectional antennas and are assumed to be located at the center of the cells. The locations of the mobiles are assumed to be uniformly distributed over the service area. The propagation characteristics have been modeled by a link gain $G_{jli} = L_{jli} / d_{jli}^{\alpha}$, where d_{jli} is the distance between the l-th mobile unit of the j-th cell and the base station in cell i. The $1/d^{\alpha}$ factor models the large scale propagation loss whereas the attenuation factors L_{jli} model the power variation due to log-normal shadow fading. The log-variance will be denoted by σ.

Outage probabilities have been obtained by means of Monte Carlo simulation for 1000 independent configurations. The value of the system protection ratio used in our simulations has been γ_0=-15dB, corresponding to a system processing gain of about 20-30 dB with typical modulation, coding and equalization schemes and performance requirements. We assume that the system has been properly designed to cope with the fast multipath fading. In all the simulations, we have assumed a voice activity factor β=0.6.

Figs. 1 and 2 show the performance of the DS/CDMA-SRA and DS/CDMA-SMIRA procedures, when n *unusable* links are removed. As can be seen from the graphs, the capacity of the system, measured in users per cell, increases rapidly as *unusable* links are removed according to the removal criterion. It can also be seen that a system using the DS/CDMA-SMIRA algorithm can achieve a capacity gain that increases with n over a system using DS/CDMA-SRA.

The performance of the DS/CDMA-DBSMIRA algorithm when n *unusable* links are removed is shown in Figs. 3 and 4. The maximum number of balancing steps m=2 and 10 are covered in these graphs. If we compare the performance of the distributed balancing algorithm to the performance of the centralized algorithms, it is obvious that the distributed scheme outperforms the SRA system and that it provides a fairly good approximation to the SMIRA algorithm. Fig. 5 shows the capacity of the system at the 10% outage probability level as a function of the maximum number of allowed balancing steps and with the maximum number of *unusable* links removed as parameter. From the figure, it can be seen that the proposed scheme converges very fast, with very few balancing steps being required for the minimum CIR to approach γ^{*}.

VII. CONCLUSIONS

In this paper, based on some results from the theory of positive matrices, we have derived the *maximum* carrier-to-adjacent channel interference achievable in a DS/CDMA cellular mobile radio system. A centralized power control algorithm, optimum in the sense that it minimizes the outage probability for a given system protection ratio, has been described. In order to overcome the practical problem produced by the computational complexity of the optimum algorithm, sequential procedures that approximate its behavior have been proposed. Numerical Monte-Carlo simulation results show the potentially large capacity gains that can be obtained by using transmitter power control in order to achieve a balanced CIR system. The use of these centralized algorithms in a cellular mobile radio network, however, would require reliable measurements of the gains in all propagation links in the system. Therefore, these algorithms serve mainly as tools to derive upper bounds on the performance of transmitter power control algorithms rather than being suited for practical implementation. Anyway, we have shown that a distributed discrete time power control algorithm can approximate the behavior of the optimum centralized algorithm by using only limited path gain information.

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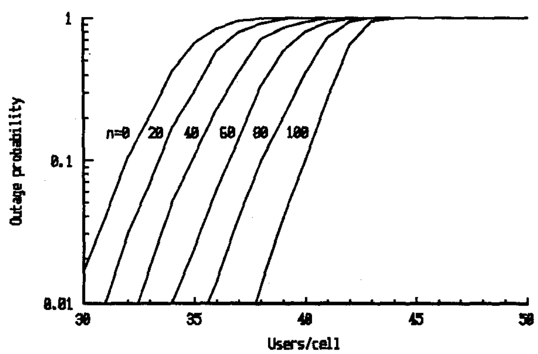

Fig. 1 Outage probability for DS/CDMA-SRA with n (maximum number of *unusable* links removed) as parameter. (α=4, σ=8dB, β=0.6)

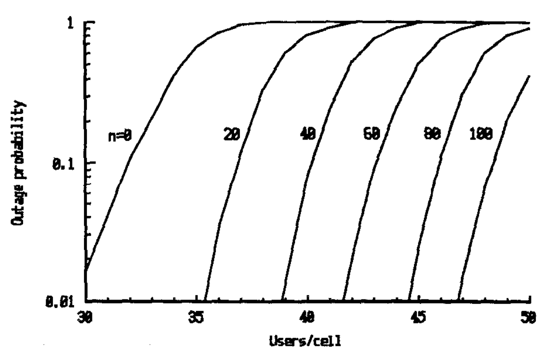

Fig. 2 Outage probability for DS/CDMA-SMIRA with n as parameter. (α=4, σ=8dB, β=0.6)

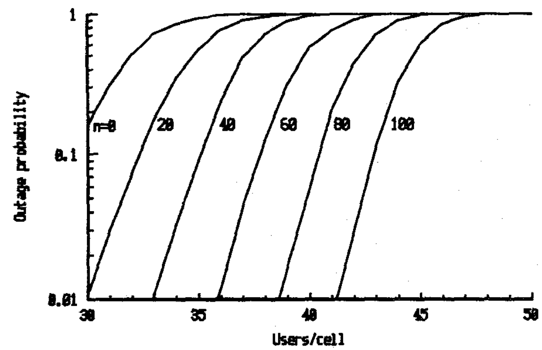

Fig. 3 Outage probability for DS/CDMA-DBSMIRA for m=2 and with n as parameter. (α=4, σ=8dB, β=0.6)

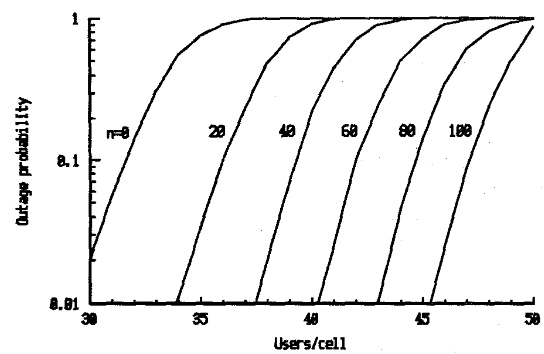

Fig. 4 Outage probability for DS/CDMA-DBSMIRA for m=10 and with n as parameter. (α=4, σ=8dB, β=0.6)

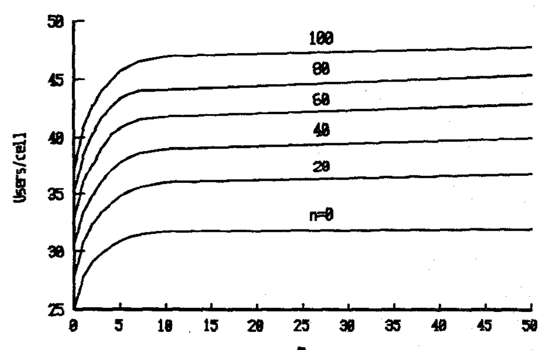

Fig. 5 System capacity at the 10% outage probability as a function of m and with n as parameter. (α=4, σ=8dB, β=0.6)