

# PERFORMANCE OF ADAPTIVE BAYESIAN EQUALIZERS IN OUTDOOR ENVIRONMENTS

José Luis VALENZUELA, Fernando CASADEVALL

Departament de Teoria del Senyal i Comunicacions

E.T.S.E.B Universitat Politècnica de Catalunya

Apdo. 30002, 08080 Barcelona, SPAIN Tel. +34 3 401 59 48 FAX. +34 3 401 72 00

E-Mail: valens@xaloc.upc.es

Abstract -Outdoor communications are affected by multipath propagation that imposes an upper limit on the system data rate and restricts possible applications. In order to overcome the degrading effect introduced by the channel, conventional equalizers implemented with digital filters have been traditionally used. In this paper a new approach based on neural networks is considered. In particular, the behavior of the adaptive Bayesian equalizer implemented by means of Radial Basis functions applied to the channel equalization of radio outdoor environments has been analyzed. The method used to train the equalizer coefficients is based on a channel response estimation. We compare the results obtained with three channel estimation methods: The least sum of square errors (LSSE) channel estimation algorithm, recursive least square (RLS) algorithm employed only to obtain one channel estimation and, finally, the RLS algorithm used to estimate the channel every decided symbol for the whole frame.

## I. INTRODUCTION

Reliable data transmission in mobile digital communication systems often requires the use of channel equalization in order to compensate for the intersymbol interference (ISI) induced by the multipath propagation. Traditionally, decision-feedback equalizers (DFE) implemented with FIR filters and maximum likelihood estimation (MLSE) have been employed for this purpose. Equalizer design for third generation mobile systems and, in particular, systems that use an advanced TDMA mobile access for Universal Mobile Telecommunication System (UMTS), must overcome problems that are much more complex than those encountered in the actual second generation.

MLSE equalization is optimum in the sense that it obtains the best performance by detecting the entire transmitted sequence that minimizes the squared error. However, its high complexity makes it useless in many practical implementations. Moreover, it has been proved that MLSE equalization has important degradation for time-varying channels [1].

Symbol-decision equalizers have been commonly implemented with DFE structures based on the linear filter

approach. These structures are simpler than MLSE despite a certain performance loss in a static environment.

The optimal solution for the symbol-decision equalizers requires nonlinear bounds that can be derived adopting a Bayesian approach, also known as the *maximum a posteriori* (MAP) equalizer. The Bayesian could be implemented with or without a decision-feedback structure. It has been shown repeatedly that feedback structures perform better than no- feedback structures. Recently, the application of some new equalization structures such as Multilayer Perceptron (MLP) and Radial Basis Function (RBF) have been in order to implement the nonlinear bounds. The equalizers implemented with MLP structures could achieve nonlinear decision bounds, although the algorithms, such as the backpropagation algorithm, used to reach the net coefficients need a great number of samples [2] and an immense computational effort and this makes them improper for equalizing mobile environments.

On the other hand, RBF structures have been presented in recent papers as effective equalizers in static channels with two main algorithms for updating the coefficients. The clustering approach [3] identifies these coefficients using a clustering algorithm. This algorithm is computationally very simple and effective for nonlinear channel distortion. However, as the backpropagation algorithm for MLP, it needs a large amount of training samples. The second approach estimates the channel model and calculates the RBF coefficients. The main advantages of this method are the short training sequences needed and the possibility of employing adaptive channel estimators that allow the results obtained with a non-adaptive algorithm to be outperformed.

In this paper we present the results obtained with the RBF structures in a time-varying channel, using different channel estimators, and compare them with the results obtained using a conventional decision-feedback equalizer. The system performance is evaluated by means of the bit error rate (BER) as a function of the signal-to-noise ratio (SNR) and the normalized delay spread [4] for several mobile speeds.

## II. TRANSMISSION MODEL

Fig. 1 shows the low-pass equivalent model of the transmission system.

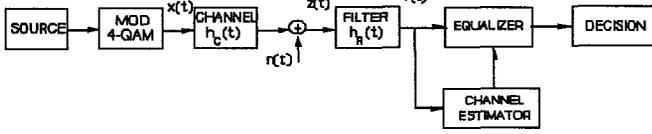


Fig. 1 Low-pass transmission model.

In general the transmitted baseband QAM signal can be formulated as:

$$s(t) = \sum_{k=-\infty}^{\infty} (a_k + jb_k) \delta(t - kT) = \sum_{k=-\infty}^{\infty} c_k \delta(t - kT) \quad (1)$$

where  $\{a_k\}$ ,  $\{b_k\}$  are independent data sequences for the in-phase and quadrature channel. These data sequences take their values from the set  $\pm 1, \pm 3, \dots, \pm(M^{1/2}-1)$  with  $M=4$  for 4-QAM. The overall filtering transfer function  $H_N(f) = H_T(f)H_R(f)$  is a raised-cosine type with a roll-off factor,  $\beta$  equal to 0.5, which is equally split between the transmitter and the receiver. The function  $H_c(f)$  models the channel behavior that introduces selective fading in the radio link. The received signal can be expressed as:

$$r(t) = \sum_{k=-\infty}^{\infty} c_k h(t - kT) + n_f(t) \quad (2)$$

where  $n_f(t)$  is the Gaussian filtered noise added by the channel and  $h(t)$  is the overall impulse response given by:

$$h(t) = h_T(t) * h_c(t) * h_R(t) = \mathcal{F}^{-1}\{H_N(f)\} * h_c(t) \quad (3)$$

where

$$\mathcal{F}^{-1}\{H_N(f)\} = h_N(t) = \frac{\text{sen}(\pi t/T) \cos(\beta \pi t/T)}{\pi t/T \sqrt{1 - (2\beta t/T)^2}} \quad (4)$$

being  $\mathcal{F}^{-1}$  the inverse Fourier transform operator,  $\beta$  the roll-off factor of the raised-cosine function and  $T$  is the symbol period.

In this paper, we have considered an outdoor non-static channel model based on COST-207 document [5,6]. In particular we have used the model denominated Typical Urban environment. This document characterizes the channel by

means of the Power Delay Profile, the number of discrete coefficients, the delay coefficient and the Rayleigh distributed amplitude of each tap, varying according to a Doppler spectrum. In this channel model three Doppler spectrum classes are considered.

The received and sampled signal at the input of the equalizer can be formulated as:

$$r(nT) = \sum_{k=-\infty}^{\infty} c_k h((n-k)T) + n_f(nT) \quad (5)$$

where  $n_f(nT_m)$  is a complex Gaussian random uncorrelated process with variance equal to  $\sigma_n^2$  and  $h[(n-k)T]$  is the sampled response of the channel model given by:

$$h[(n-k)T] \equiv \sum_{i=1}^D (h_{di} + jh_{qi}) h_N[(n-k)T - \tau_i] \quad (6)$$

where  $\tau_i$  is the delay of the channel tap  $i$  defined in COST 207 document,  $(h_{di} + jh_{qi})$  is the instantaneous complex amplitude of the tap  $i$  whose mean power and time variation are also defined in COST 207 document and being

$$h_N[(l-k)T] = \frac{\text{sen}(\pi \gamma) \cos(\beta \pi \gamma)}{\pi \gamma \sqrt{1 - (2\beta \gamma)^2}} \quad (7)$$

with

$$\gamma = (l-k) \quad (8)$$

## III. BAYESIAN DECISION-FEEDBACK EQUALIZER

The MAP equalizer reaches the optimum performance in a symbol-decision system based on a probabilistic decision algorithm. For a finite sampled linear impulse response  $h(kT)$ , the received symbols without noise are:

$$r_k = \sum_{i=-p}^q d_i h_{k-i} \quad (9)$$

For an input sequence with length equal to  $N$ , the algorithm decides the symbol with higher probability:

$$\text{MAX}_{d_n} \{P(d_n | r_1, \dots, r_N) \quad n=1, \dots, M\} \quad (10)$$

It is clear that for a modulation with  $M$  symbols there are  $M^L$  possible states, with  $L=p+q+1$ . Then, when the decided symbol corresponds to the central impulse response sample, the possible states at instant  $k$  are:

$$\sigma_k = (d_{k+p}, \dots, d_k, \dots, d_{k-q}) \quad (11)$$

The Bayes theorem could be applied to equation (11) giving the following expression:

$$P(d_k | r_1, \dots, r_N) = \frac{\sum_{\sigma_k \in \Omega(d_k)} P(\sigma_k, r_1, \dots, r_N)}{\sum_{\sigma_k \in \Omega_0} P(\sigma_k, r_1, \dots, r_N)} \quad (12)$$

where  $\Omega_0$  is the set that encompasses all  $M^L$  states and  $\Omega(d_k)$  is the set of states when the symbol  $k$  is  $d_k$ . Then, each term of equation (13) could be factorized as:

$$P(\sigma_k, r_1, \dots, r_N) = P(\sigma_k, r_1, \dots, r_{k-1}) P(r_k, \dots, r_N | \sigma_k) = \quad (13)$$

with

$$P(\sigma_k, r_1, \dots, r_{k-1}) = \sum_{\sigma_{k-1}} P(\sigma_{k-1}, r_1, \dots, r_{k-2}) e^{-\lambda(\zeta_{k-1})} \quad (14)$$

$$P(r_k, \dots, r_N | \sigma_k) = \sum_{\sigma_{k+1}} P(r_{k+1}, \dots, r_N | \sigma_{k+1}) e^{-\lambda(\zeta_k)}$$

and

$$e^{-\lambda(\zeta_k)} = e^{-|z_k - \sum_{i=1}^p h_i \hat{d}_{k,i}|^2} \quad (15)$$

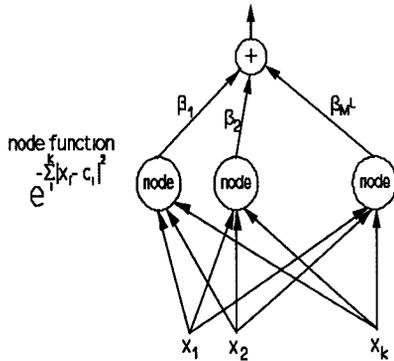


Fig. 2 Radial Basis Function net.

Comparing this set of equations with the RBF structure shown in Fig. 2, it is clear that by calculating the centers ( $c_i$ ) of each node as the noiseless estimated received signal, the net could perform as a MAP algorithm.

## IV. CHANNEL ESTIMATORS

In this paper we have evaluated three kinds of channel estimators. First, we have used the LSSE algorithm. This algorithm is similar to the estimator used in the GSM system based on a correlation procedure with a training sequence and has been chosen for the Universal Mobile Telecommunication System (UMTS) [7,8]. The main characteristic of this algorithm is the low computational cost required to estimate the impulse response. However, this algorithm cannot be implemented in an adaptive form.

The second simulated system uses the classical RLS in order to calculate the impulse response. When the estimated response is obtained at the end of the training sequence, the algorithm is stopped. This algorithm needs shorter training sequences than the LSSE algorithm. It only needs  $2N-1$  where  $N$  is the length of the estimated impulse response [9], but it has a higher computational cost. This algorithm is more appropriate than the least mean square (LMS) algorithm because of the low-time convergence achieved with LMS which produces a loss of tracking in fast varying channels [4].

We have also evaluated the system performance utilizing the same RLS algorithm but using the decided symbol when the training sequence is finished. This technique compensates for the time variation effects of the mobile channel.

In the first and second systems, the equalizer coefficients are calculated when the training sequence is finished and the equalizer keeps these coefficients for the whole information frame. However, the third system needs to calculate the coefficients every iteration.

Finally we have considered a frame with 20 training symbols and 100 information symbols when the LSSE estimator is employed, and 10 training symbols and 110 information symbols when the RLS estimator is used.

## V. RESULTS

We have examined the effectiveness of the RBF used as an equalizer in fighting the multipath and fading introduced by the radio channel. The criterion used to evaluate the system quality is the bit error rate,  $P_e$ , versus the signal-to-noise ratio and the normalized delay spread. The equalizers have the following characteristics: the RBF net has been implemented with 3 input signal nodes and 3 feedback-decided symbol nodes. The conventional DFE has the same structure in order to have the same information in each equalizer. The equalizer coefficients are calculated from an estimated impulse response of 5 taps. The environment modeled is a typical urban channel with 12 taps and 1 microsecond of delay spread [5,6].

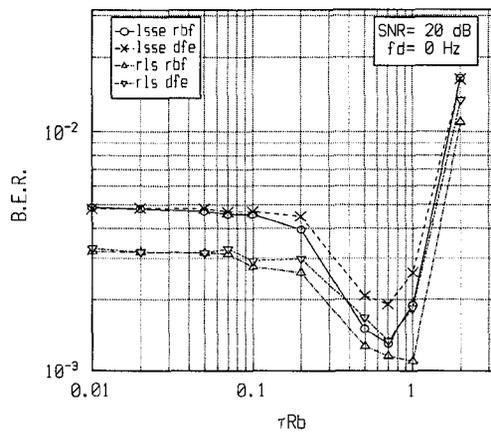


Fig. 3. B.E.R. versus normalized delay spread in a static environment.

In Fig. 3, we show the evolution of the mean value of the bit-error probability against the normalized delay spread value,  $\tau T_b$ ,  $T_b$  being the bit period, 20 dB of signal-to-noise ratio, and a static environment. In the figure, the results obtained for both equalizers, RBF and conventional DFE, are compared. Their coefficients have been calculated taking the LSSE algorithm and RLS algorithm as channel estimators. It could be noticed that equalizers trained with the LSSE algorithm have a performance loss for all the values of the normalized delay spread. It is also shown that if the delay spread is higher than 0.2, the RBF performance is better than the conventional DFE. These results seem to be reasonable, since a RBF structure is able to create more complicated decision regions in this case as they are needed.

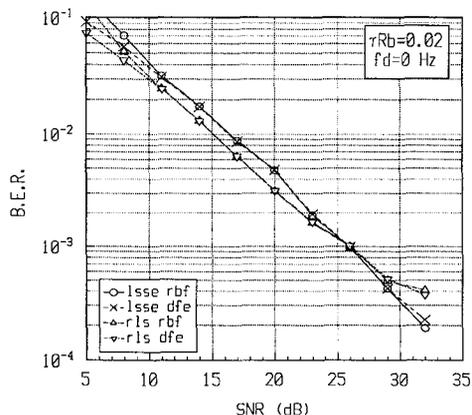


Fig. 4. B.E.R. versus SNR in a low dispersive and static environment.

In Figs. 4 and 5 we show the evolution of the BER versus the signal-to-noise ratio considering a null frequency Doppler. In these graphics, the effectiveness of the RLS algorithm compared to the LSSE algorithm is shown. It could be noticed that a gain of 2 dB up to a SNR of 25 dB for a

normalized delay spread equal to 0.02 is obtained. The system with the RLS algorithm also has a gain of about 1 or 2 dB for a normalized delay spread equal to 1.0. On the other hand, it is shown that the RBF equalizer is better than a conventional DFE for a normalized delay spread equal to 1.0, obtaining a gain approximately of 1 dB.

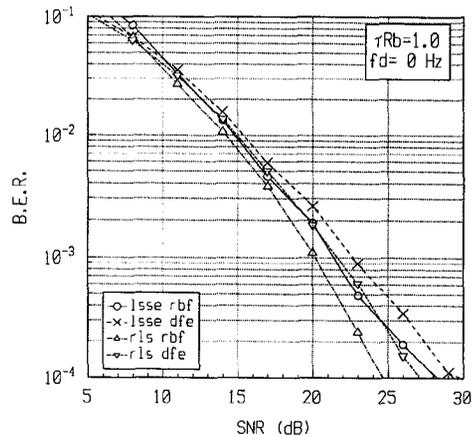


Fig. 5. B.E.R. versus SNR in a high dispersive and static environment.

When we consider the Doppler effect (Figures 6 to 9), it could be noticed that the system has a great loss of performance, especially for low values of the normalized delay spread. It is shown in Figure 6 that only when tracking RLS is used, (in Figures tk. rls), does the bit error descend to approximately  $10^{-2}$ . In these figures it could be noticed that when the Doppler effect is considered, the LSSE estimator performs better than the no-tracking RLS estimator, (in Figure no tk. rls), for low values of the normalized delay spread.

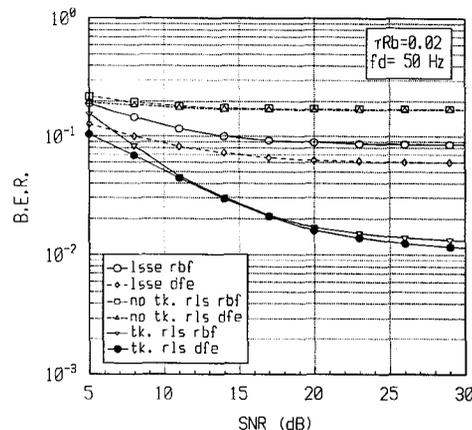


Fig. 6. B.E.R. versus SNR in a low dispersive environment with 50Hz of Doppler.

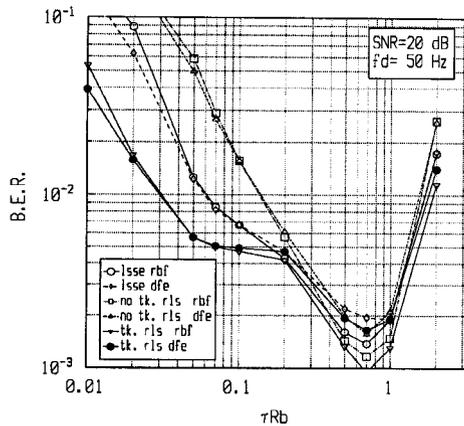


Fig. 7. B.E.R. versus normalized delay spread with 50 Hz of Doppler.

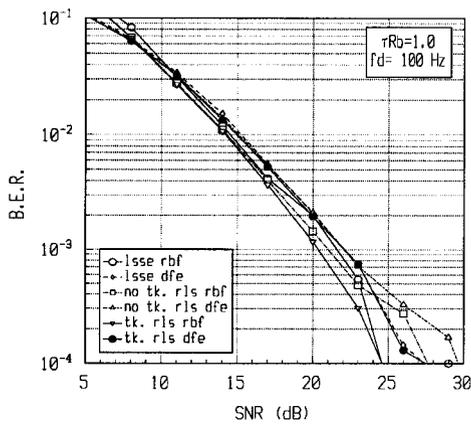


Fig. 8. B.E.R. versus SNR in a high dispersive environment with 100Hz of Doppler.

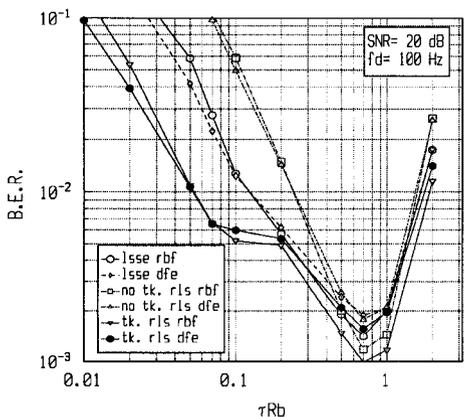


Fig. 9. B.E.R. versus delay spread with 100 Hz of Doppler.

## VI. CONCLUSIONS

In this paper the performance of the MAP algorithm implemented with RBF using three estimators in a typical

urban radio environment have been analyzed and compared with a conventional DFE. It has been shown that the MAP algorithm offers effectiveness for adaptive equalization for the three analyzed channel estimators with respect to the conventional DFE and that it provides a viable solution to the problem of channel distortion in digital communication systems. It has been shown that this new technique allows the signal-to-noise ratio in high dispersive channels to be reduced more than 1 dB. On the other hand, it has been proved that the LSSE estimator produces losses in performance for low dispersive static channels. However, it is more robust respect to the no-tracking RLS algorithm when the doppler effect is taken into account. This effect is produced because the simulated system with the LSSE estimator has a longer training sequence. When the no-tracking RLS algorithm has the same length of training sequence, we have proved that it has the same behavior as the LSSE algorithm. However, this does not produce the same effect for tracking RLS algorithms. We think that in this paper the necessity of using RLS tracking techniques to compensate the effects produced by time-varying dispersive channels has been demonstrated.

## VII. ACKNOWLEDGMENTS

The work described in this paper has been carried out within the CICYT TIC94-0870-C02-01 project, in the framework of National Plan of Spain.

## VIII. REFERENCES

- [1] S. Chen, S. McLaughlin, B. Mulgrew, P. M. Grant, "Adaptive Bayesian Decision Feedback Equalizer for Dispersive Mobile Radio Channels", IEEE Trans. on Comm., vol. 43, no. 5, pp. 1937-1945, May 1995.
- [2] M. Peng, C. L. Nikias y J. G. Proakis, "Adaptive Equalization with Neural Networks: New Multilayer Perceptron Structures and their Evaluation", IEEE-ICASSP-92, vol. 2, San Francisco, March 1992.
- [3] S. Chen, B. Mulgrew, P. M. Grant, "A Clustering Technique for Digital Communications Channel Equalization using Radial Basis Function Networks", IEEE Trans. on Neural Net., vol. 4, pp. 570-579.
- [4] J.G. Proakis, "Digital Communications", 2nd ed. New York, McGraw-Hill, 1982.
- [5] COST 207, "Digital Land Mobile Radio Communications", Final Report. Office for Official Pub. of the European Commun., Luxembourg, 1989.
- [6] R. W. Lorentz, "Corrections of COST 207 Proposals for Simulation of Radio Channels", COST 231 TD (91) 26, Darmstadt, February 1991.
- [7] S.N. Crozier, D.D. Falconer, S.A. Mahmoud, "Least Sum of Squared Errors (LSSE) Channel Estimation", IEE Proc.-F vol.138, no. 4, pp. 371-378, August 1991.
- [8] A. Urie, M. Streeton, C. Mourot, "An Advanced TDMA Mobile Access System for UMTS" PIMRC 94, pp. 685-690, Sept. 94. The Hague.
- [9] J.G. Proakis, "Adaptive Equalization for TDMA Digital Mobile Radio", IEEE Trans. on Vehicular Tech., vol. 40, no. 2, pp. 333-341, May 1991.