Particle Swarm Optimization Assisted Multiuser Detector for M-QAM DS/CDMA Systems

Leonardo D. Oliveira, Taufik Abrão, *Member, IEEE* Paul Jean E. Jeszensky, *Senior Member, IEEE* and
Fernando Casadevall *Member, IEEE*

Abstract—This paper analyzes the particle swarm optimization multiuser detector (PSO-MuD) under high-order modulation schemes, (particularly for M-QAM), in DS/CDMA systems single-input-single-output (SISO) multipath channels. In order to avoid the computation of complex-valued variables in high-order squared modulation, the optimization problem is reformulated as a real-valued problem. Considering previous results on literature for low-order modulation formats (usually binary/quadrature phase shift keying - BPSK/QPSK), a performance×complexity trade-off comparison is carried out between PSO-MuD and local search multiuser detector (LS-MuD). Performance is evaluated by the symbol error rate (SER), and complexity by necessary number of cost function calculations for convergence. If the background for BPSK heuristic multiuser detection (HEUR-MuD) problem [1] indicates that the 1-opt local search method is enough to achieve excellent performance×complexity trade-offs, our Monte-Carlo simulation results and analysis show herein indicate the LS-MuD presents a lack of search diversity under high-order modulation formats, while the PSO-MuD is efficient to solve the MuD problem for high-order modulation schemes.

Index Terms—DS/CDMA, high-order modulation, local search optimization, PSO, sub-optimal multiuser detection.

I. INTRODUCTION

THE capacity of a DS/CDMA system in multipath channel is limited mainly by the multiple access interference (MAI), self-interference (SI), near-far effect and fading. The conventional receiver (Rake) explores the path diversity in order to reduce fading effect, but it is not able to mitigate neither the MAI nor the near-far effect [2], [3].

Multiuser detection is an alternative to deal with this limitation. The best performance is acquired by the optimum multiuser detection (OMuD), based on the log-likelihood function (LLF) [3], but its complexity is prohibitive. In the last decade, a variety of multiuser detectors with low complexity and sub-optimum performance were proposed, such as linear detectors, subtractive interference canceling [2], [3], semidefinite

programming (SDP) approach [4]–[6] and heuristic methods [1], [7]–[13].

The suboptimal multiuser detection based on semidefinite relaxation (SDR-MuD) approach and HEUR-MuD were initially proposed to work on low-order modulation, usually BPSK/QPSK signals. For instance, in SDR-MuD, the optimal maximum likelihood (ML) detection problem is carried out by relaxing the associated combinatorial programming problem into an SDP problem with both the objective function and the constraint functions being convex functions of continuous variables. Heuristic and semidefinite relaxation MuD approaches have been shown to yield near-optimal performance in BPSK/QPSK signals multiuser detection [1], [4]–[7], [9].

In four generation (4G) wideband wireless communication systems, multiple-input-multiple-output (MIMO) schemes and high-order modulation methods, such as Mary phase shift keying (M-PSK) or Mary quadrature amplitude modulation (M-QAM), are often adopted in order to increase the throughput or the system capacity. Therefore, there has been much interest recently in extending the near-optimum low-complexity detection approaches to detect high-order modulated signals [11], [12], [14]–[20]. For instance, SDR-MuD approaches, previously applied to single-input-single-output (SISO) $64-$QAM multicarrier CDMA systems [15], were applied recently to high-order QAM constellations in MIMO systems in [14], [16]–[18], with M-PSK constellation in [19], and in coded MIMO systems with pulse-amplitude modulation (PAM) constellations was investigated in [20].

Another approach to achieve near-optimum low-complexity multiuser detection with high-order symbol modulation schemes and fading channels consists in adopt heuristic procedures. However, there are few works dealing with high-order modulation HEUR-MuD SISO or MIMO [11], [12] systems.

This work analyzes two heuristic techniques applied to near-optimum asynchronous DS/CDMA multiuser detection problem under high-order modulation formats and SISO multipath channels. Previous results on literature [1], [8], [10], [13], [21] suggest that evolutionary algorithms and particle swarm optimization have similar performance, and that a simple local search heuristic optimization is enough to solve the MuD problem with low-order modulation (BPSK [1] and QPSK). However, for high-order modulation formats, the LS-MuD does not have a good performance due to a lack of search diversity, whereas the PSO-MuD is shown to be more efficient for solving the optimization problem under M-QAM modulation. For the best of our knowledge, no previous results were found in the literature applying PSO, LS or evolutionary approaches to M-QAM MuD problem.

The paper is organized as follows. The M-QAM DS/CDMA model system is revised in Section II. For squared high-order modulation, the optimization procedure is described in Section III as an alternative LLF containing only real values. The PSO and LS optimization procedures based on real-valued LLFs, as well as parameters choices, are discussed in Section IV. A performance×complexity trade-off analysis is pointed out in Section V, in order to evaluate the feasibility of these two heuristic techniques in solving efficiently the M-QAM MUD problem. Finally, the main conclusions are highlighted in Section VI.

II. SYSTEM MODEL

Let us consider an asynchronous SISO DS/CDMA system under fading channels and shared by K users. The base-band transmitted signal of the kth user, containing I M-QAM symbols, is [22]:

$$\underline{s_k}(t) = \sqrt{\frac{\mathcal{E}_k}{T}} \sum_{i=1}^{I} s_k^{(i)} g_k(t - iT), \tag{1}$$

where $\mathcal{E}_k = A_k^2 T = (A_{k,\text{real}}^2 + A_{k,\text{imag}}^2)T$ is the symbol energy; the subscript real and imag indicate in-phase and quadrature amplitude components, and T is the symbol duration. Each symbol $s_k^{(i)}$, $k = 1, \ldots, K$ is taken independently and with equal probability from a complex alphabet set $\mathcal{A}$ of cardinality $M = 2^m$ in a squared constellation points, i.e., $s_k^{(i)} \in \mathcal{A} \subset \mathbb{C}$. Figure 1 shows some modulation formats. The normalized spreading sequence for the k-th user is given by:

$$g_k(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_k(n) p(t - nT_c), \qquad 0 \le t \le T, \tag{2}$$

where $a_k(n)$ is a pseudo-noise sequence with N chips assuming the values $\{\pm 1\}$, $p(t)$ is the pulse shaping, assumed rectangular with unitary amplitude and duration T_c, with T_c being the chip interval. The processing gain is given by $N = T/T_c$, and is illustrated in Figure 2.
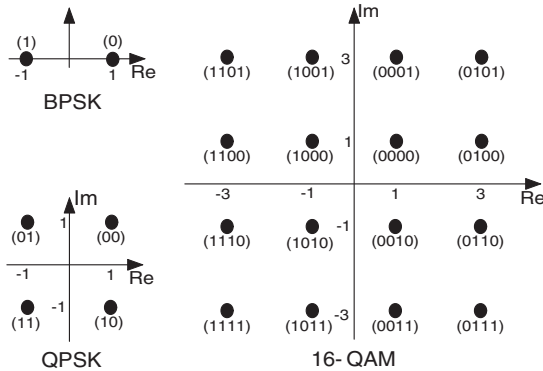

Fig. 1.   Some modulation formats with Gray mapping.

The received signal containing I symbols for each user in multipath fading channel can be expressed by:

$$r(t) = \sum_{i=0}^{I-1} \sum_{k=1}^{K} \sum_{\ell=1}^{L} A_k s_k^{(i)} g_k(t - iT - \tau_{k,\ell}) \zeta_{k,\ell}^{(i)} + \eta(t), \tag{3}$$

where L is the number of paths fading, admitted equal for all K asynchronous users, $\tau_{k,\ell}$ is the total delay for the signal of the kth user, ℓth path; $\eta(t)$ is the additive white Gaussian noise (AWGN) with bilateral power spectral density equal to $N_0/2$, and $\zeta_{k,\ell}^{(i)}$ is the channel complex coefficient for the kth user, ℓth path and ith symbol, defined as:

$$\zeta_{k,\ell}^{(i)} = \gamma_{k,\ell}^{(i)} e^{j\phi_{k,\ell}^{(i)}}, \tag{4}$$

where the module $\gamma_{k,\ell}^{(i)}$ is a random variable characterized by a Rayleigh distribution and the phase $\phi_{k,\ell}^{(i)}$ is a random variable modeled by the uniform distribution $\mathcal{U}(0, 2\pi)$. A slow and frequency selective channel[1] is assumed.
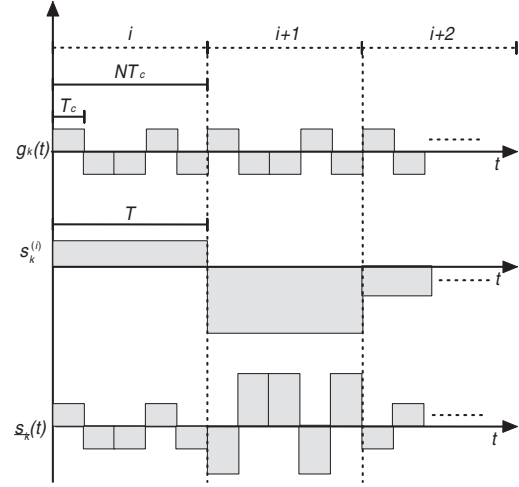

Fig. 2.   Standard spreading with PAM signal and $N = 5$.

At the base station receiver (uplink), using D branches diversity per user, the received signal is despread by a matched filter bank, with a total of KD matched filter branches, and $D \le L$. After despreading, the ith symbol on the ℓth path of the kth user is given by

$$\begin{aligned} y_{k,\ell}^{(i)} &= \frac{1}{T} \int_{iT}^{(i+1)T} r(t) g_k(t - \tau_{k,\ell}) dt \\ &= A_k \gamma_{k,\ell}^{(i)} s_k^{(i)} + SI_{k,\ell}^{(i)} + I_{k,\ell}^{(i)} + \eta_{k,\ell}^{(i)}. \end{aligned} \tag{5}$$

The first term is the signal of interest, the second corresponds to the self-interference, the third to the MAI and the last one corresponds to the filtered AWGN.

The cross-correlation element between the kth user, ℓth path and uth user, dth path can be identified by:

$$\rho_{k,\ell,u,d} = \frac{1}{T} \int_0^T g_k(t - \tau_{k,\ell}) g_u(t - \tau_{u,d}) dt. \tag{6}$$

After combining D output correlators for each user and applying a hard decisor, the estimate for the symbols vector can be denoted as $\widehat{\mathbf{s}} = [\widehat{s}_1 \ \widehat{s}_2 \ \ldots, \ \widehat{s}_K]^T$. Figure 3 illustrates the SISO uplink M-QAM DS/CDMA heuristic multiuser detection problem.

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[1] Slow channel: channel coefficients were admitted constant along the symbol period T; and frequency selective condition is hold: $\frac{1}{T_c} >> (\Delta B)_c$, the coherence bandwidth of the channel.
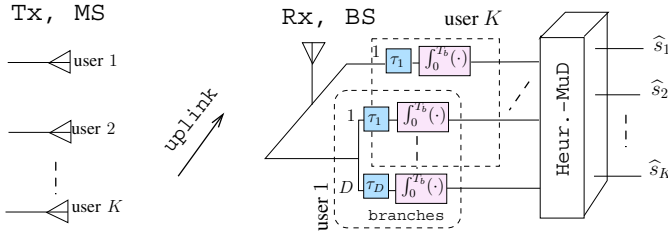
Fig. 3. SISO uplink DS/CDMA HEUR-MUD. Tx: transmitter antenna; Rx: receiver antenna; MS: mobile station; BS: base station.

III. OPTIMUM DETECTION

In order to deal with MAI, the OMUD based on LLF [3] estimates the symbols for all K users by choosing the symbol combination associated with the minimal distance metric among all possible symbol combinations in the $M = 2^m$ constellation points. In this work, the one-shot asynchronous channel approach is adopted, where an asynchronous scenario with K users, I symbols and D branches is equivalent to a synchronous scenario with KID virtual users.

In order to avoid the need to handle complex-valued variables in high-order squared modulation, the alphabet set can be simplified as $\mathcal{A}_{\text{real}} = \mathcal{A}_{\text{imag}} = \mathcal{C} \subset \mathbb{Z}$ of cardinality $M/2$, e.g., $16-$QAM ($m = 4$): $s_k^{(i)} \in \mathcal{C} = \{\pm 1, \pm 3\}$.

Assuming uplink asynchronous communication with K mobile users in a SISO squared-QAM DS/CDMA multipath Rayleigh channel with diversity $D \leq L$, we can define the LLF as a decoupled optimization problem with only real-valued variables:

$$\Omega(\mathbf{q}_\nu) = 2\mathbf{q}_\nu^T \mathbf{M}^T \mathbf{z} - \mathbf{q}_\nu^T \mathbf{M}\mathbf{Q}\mathbf{M}^T \mathbf{q}_\nu, \qquad (7)$$

and the vector that maximizes (7) will be the solution of the maximum likelihood function, as follows:

$$\mathbf{q}^* = \max_{\mathbf{q}_\nu} [\Omega(\mathbf{q}_\nu)] \qquad (8)$$
$$\text{s.t.} \qquad \{q_{\nu,a}\} \in \mathcal{C} \subset \mathbb{Z},$$

with definitions:

$$\mathbf{z} := \begin{bmatrix} \text{Re}\{\mathbf{y}\} \\ \text{Im}\{\mathbf{y}\} \end{bmatrix}; \quad \mathbf{M} := \begin{bmatrix} \text{Re}\{\mathbf{AH}\} & -\text{Im}\{\mathbf{AH}\} \\ \text{Im}\{\mathbf{AH}\} & \text{Re}\{\mathbf{AH}\} \end{bmatrix};$$

$$\mathbf{q}_\nu := \begin{bmatrix} \text{Re}\{\mathbf{s}_\nu\} \\ \text{Im}\{\mathbf{s}_\nu\} \end{bmatrix}; \quad \mathbf{Q} := \begin{bmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix}, \qquad (9)$$

where $\mathbf{z} \in \mathbb{R}^{2KID \times 1}$, $\mathbf{M} \in \mathbb{R}^{2KID \times 2KID}$, $\mathbf{q}_\nu \in \mathcal{C}^{2KID \times 1}$, $\mathbf{Q} \in \mathbb{R}^{2KID \times 2KID}$. The vector $\mathbf{s}_\nu$ in (9) is defined as $\mathbf{s}_\nu = [s_{\nu,1} \, s_{\nu,2}, \ldots s_{\nu,a} \ldots s_{\nu,KID}]^T$, with $s_{\nu,a} \in \mathcal{A}$ a M-QAM complex symbol. $\mathbf{H}$ and $\mathbf{A}$ are the coefficients and amplitudes diagonal matrices, $\mathbf{y} \in \mathbb{C}^{KID \times 1}$ is the correlator output vector; the Hermitian operator is defined by $(\cdot)^H = [(\cdot)^*]^T$ and the block-tridiagonal, block-Toeplitz cross-correlation matrix $\mathbf{R}$ is composed by the sub-matrices $\mathbf{R}[1]$ and $\mathbf{R}[0]$ [3]:

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}[0] & \mathbf{R}[1]^T & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} \\ \mathbf{R}[1] & \mathbf{R}[0] & \mathbf{R}[1]^T & \ldots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}[1] & \mathbf{R}[0] & \ldots & \mathbf{0} & \mathbf{0} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{R}[1] & \mathbf{R}[0] \end{bmatrix}, \quad (10)$$

with $\mathbf{R}[0]$ and $\mathbf{R}[1]$ being KD matrices with elements [3]:

$$\rho_{i,j}[0] = \begin{cases} 1, & \text{if } (k = u) \text{ and } (\ell = d) \\ \rho_{k,\ell,u,d}, & \text{if } (k < u) \text{ or } (k = u \text{ and } \ell < d) \\ \rho_{u,d,k,\ell}, & \text{if } (k > u) \text{ or } (k = u \text{ and } \ell > d), \end{cases}$$

$$\rho_{i,j}[1] = \begin{cases} 0, & \text{if } k \geq u \\ \rho_{u,d,k,\ell}, & \text{if } k < u \end{cases}, \qquad (11)$$

where $i = (k-1)D + \ell$, $j = (u-1)D + d$ and $k, u = 1, 2, \ldots, K$; $\ell, d = 1, 2, \ldots, D$, where cross-correlation elements are obtained by (6).

The vector $\mathbf{q}_\nu$ in (9) belongs to a discrete set with size depending on M, K, I and D. Hence, (8) can be solved directly using a $m-$dimensional ($m = \log_2 M$) search method. Therefore, the associated combinatorial problem in an exhaustive search fashion having an exponential computational complexity that becomes prohibitive even for a moderate product $MKID$, i.e., even for a $16-$QAM modulation format and a moderate number of users K, symbols I and correlators D.

In the next section, PSO and LS heuristic approaches will be explored in order to solve (7) efficiently.

IV. SISO M-QAM DS/CDMA HEUR-MUD SUB-OPTIMAL APPROACHES

A. PSO M-QAM Multiuser Detector

The PSO principle is the movement of a group of particles, randomly distributed in the search space, each one with its own position and velocity. The position of each particle is modified by the application of velocity in order to reach a better performance [23]. The interaction among particles is inserted in the calculation of particle velocity. This work adopts a binary PSO [24]; hence, each candidate-vector defined like $\mathbf{q}_\nu[t]$ has its binary representation, $\check{\mathbf{q}}_i[t]$, of size $mKID$, used for the velocity calculation:

$$\mathbf{v}_i[t+1] = \omega \cdot \mathbf{v}_i[t] + \phi_1 \cdot \mathbf{U}_{i_1}[t](\check{\mathbf{q}}_i^{\text{best}}[t] - \check{\mathbf{q}}_i[t])$$
$$+ \phi_2 \cdot \mathbf{U}_{i_2}[t](\check{\mathbf{q}}_g^{\text{best}}[t] - \check{\mathbf{q}}_i[t]), \qquad (12)$$

where ω is the weight of the previous velocity in the present speed calculation; $\mathbf{U}_{i_1}[t]$ and $\mathbf{U}_{i_2}[t]$ are diagonal matrices with dimension $mKID$, and elements are random variables with uniform distribution $\mathcal{U} \in [0, 1]$, generated for the ith particle at instant t; $\check{\mathbf{q}}_g^{\text{best}}[t]$ and $\check{\mathbf{q}}_i^{\text{best}}[t]$ are the best global position and the best local positions found until the tth iteration, respectively; ϕ_1 and ϕ_2 are weight factors regarding the best particles and the best global positions influences in the velocity update, respectively.

For MUD optimization problem, each element of $\check{\mathbf{q}}_i[t]$ in (12) just assumes "0" or "1" values. The adjustment of velocity in order to move particle in a discrete mode is accomplished by the sigmoid function [24]:

$$S(v_{ik}[t]) = \frac{1}{1 + e^{-v_{ik}[t]}},$$

where $v_{ik}[t]$ is the kth element of the ith particle velocity, and the selection of the future particle position is obtained through the statement:

$$\text{if } \mathrm{u}_{ik}[t] < S(v_{ik}[t]), \quad \check{q}_{ik}[t+1] = 1;$$
$$\text{otherwise}, \quad \check{q}_{ik}[t+1] = 0, \qquad (13)$$

where $\check{q}_{ik}[t]$ is the kth element of $\check{\mathbf{q}}_i[t]$ and $u_{ik}[t]$ is a random variable with uniform distribution $\mathcal{U} \in [0,1]$.

1) PSO parameters optimization: Fast convergence without losing certain exploration and exploitation capabilities could be obtained increasing ϕ_2 parameter in relation to the standard values adopted in the literature: e.g., in [10], simulation experiments for BPSK MuD problem showed that $\phi_2 \in [2; 10]$ (while $\phi_1 = 2$) had a positive impact on the search intensification for the best global position. Table I illustrates different performance achieved combining ϕ_1 and ϕ_2 values for a system with medium loading and medium received signal-noise ratio (SNR). Hence, considering the performance×complexity trade-off, it was adopted in this work $\phi_1 = 2$ and $\phi_2 = 10$.

TABLE I
COARSE OPTIMIZATION RESULTS FOR ϕ_1 AND ϕ_2 PARAMETERS.

(ϕ_1, ϕ_2)	(2,10)	(2,6)	(10,2)	(2,2)	(2,1)	(1,1)
SER $[\times 10^{-2}]$	8.9	8.9	8.7	8.7	8.6	8.7
Iterations	68	70	90	92	114	115

In order to obtain further diversity for the search universe, the $V_{\max}$ factor is added to the PSO model, which will be responsible for limiting the velocity in the range $[\pm V_{\max}]$. The insertion of this factor in the velocity calculation enables the algorithm to escape from possible local optima. The likelihood of a bit change increases as the particle velocity crosses the limits established by $[\pm V_{\max}]$, as shown in Table II. For BPSK PSO-MuD algorithm, the better performance×complexity trade-off was obtained in [10] setting $V_{\max} = 4$. Herein, for 16-QAM PSO-MuD problem, simulation results also indicated a best maximal velocity factor value of $V_{\max} = 4$.

TABLE II
MINIMUM BIT CHANCE PROBABILITY AS A FUNCTION OF $V_{\max}$.

$V_{\max}$	1	2	3	4	5
$1 - S(V_{\max})$	0.2690	0.1192	0.0474	0.0180	0.0067

For the inertial weight, ω, it was observed that high values imply in fast convergence, but this means a lack of search diversity, and the algorithm can easily be trapped in some local optimum, whereas a small value for ω results in a slow convergence due to excessive bit changes. In this work, it was adopted $\omega = 1$. Table III offers some insight on this parameter optimization process.

TABLE III
OPTIMIZATION RESULTS FOR ω PARAMETER.

ω	0.50	0.75	1.00	1.25	1.50	2.00	5.00
SER $[\times 10^{-1}]$	n.c.	3.50	0.75	0.85	0.90	1.00	3.00

n.c.: non convergence.

Algorithm 1 describes the implemented M-QAM PSO-MuD multiuser detector. Population size $\mathcal{P}$ is modified taking into account the background in [10] , in order to obtain efficiency in M-QAM optimization scenarios.

B. Local Search M-QAM Multiuser Detector

The local search is an optimization method that consists of searches in a previously established neighborhood [25].

Algorithm 1 M-QAM PSO-MuD

Input: $\mathbf{q}^{\text{MFB}}$, $\mathcal{P}$, G, ω, ϕ_1, ϕ_2, $V_{\max}$;　　　**Output:** $\mathbf{q}^*$
begin
1.　initialize first population: $t = 0$;
　　$\mathbf{Q}[0] = \mathbf{q}^{\text{MFB}} \cup \widetilde{\mathbf{Q}}$, where $\widetilde{\mathbf{Q}}$ contains $(\mathcal{P}-1)$ particles randomly generated;
　　$\mathbf{q}_i^{\text{best}}[0] = \mathbf{q}_i[0]$ and $\mathbf{q}_g^{\text{best}}[0] = \mathbf{q}^{\text{MFB}}$;
　　$\mathbf{v}_i[0] = \mathbf{0}$: null initial velocity;
2.　while $t \leq G$
　　a. calculate $\Omega(\mathbf{q}_i[t])$, $\forall \mathbf{q}_i[t] \in \mathbf{Q}[t]$ using (7);
　　b. update velocity $\mathbf{v}_i[t]$, $i = 1, \ldots, \mathcal{P}$, through (12);
　　c. update best positions:
　　　for $i = 1, \ldots, \mathcal{P}$
　　　　if $\Omega(\mathbf{q}_i[t]) > \Omega(\mathbf{q}_i^{\text{best}}[t])$, $\mathbf{q}_i^{\text{best}}[t+1] \leftarrow \mathbf{q}_i[t]$
　　　　else $\mathbf{q}_i^{\text{best}}[t+1] \leftarrow \mathbf{q}_i^{\text{best}}[t]$
　　　end
　　　if $\exists~ \mathbf{q}_i[t]$ such that $\left[\Omega(\mathbf{q}_i[t]) > \Omega(\mathbf{q}_g^{\text{best}}[t])\right] \wedge$
　　　　　　　　　　$\left[\Omega(\mathbf{q}_i[t]) \geq \Omega(\mathbf{q}_j[t])\right], j \neq i$,
　　　　$\mathbf{q}_g^{\text{best}}[t+1] \leftarrow \mathbf{q}_i[t]$
　　　　else $\mathbf{q}_g^{\text{best}}[t+1] \leftarrow \mathbf{q}_g^{\text{best}}[t]$
　　d. Evolve to a new swarm population $\mathbf{Q}[t+1]$, using (13);
　　e. set $t = t + 1$.
　　end
3.　$\mathbf{q}^* = \mathbf{q}_g^{\text{best}}[G]$.
end

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$\mathbf{q}^{\text{MFB}}$: Rake output, replaced in real form.
$\mathcal{P}$: Population size.
G: number of swarm iterations.

For the local search algorithm it is important to restrict the neighborhood and to choose a good start point in order to find a valid solution with low complexity.

The degree of $k-$opt local search (k-opt LS) can be chosen considering m bits mapping a symbol that belongs to a constellation of size $M = 2^m$ symbols, and it is naturally advantageous to use Gray mapping. The search complexity increases exponentially with k, and it becomes too computationally expensive to adopt $k \geq 2$ for high-order modulation ($M \geq 16$) and large number of users. Considering that in [1] the 1-opt LS had been solved efficiently the SISO DS/CDMA MuD problem with BPSK modulation, for comparison purposes, herein the M-QAM 1-opt LS-MuD performance×complexity was evaluated. The 1-opt LS algorithm evaluates each unitary Hamming distance candidate-vector from the current best solution (obtained from previous iterations) [1], [13]. If a better candidate-vector is found, the current best solution is updated, and a new iteration takes place. The search terminates when there is no improvement. The steps of the implemented conventional 1-opt LS-MuD are described in Algorithm 2.

Basically, three advantages make the 1-opt LS algorithm a natural choice in order to reach an efficient solution for the MuD problem: a) absence of input parameters; b) simple stop criterion, avoiding prior calculation; c) simple strategy, with possible additional simplifications.

The "feasible unitary Hamming distance neighbor" in step 2.a. of Algorithm 2 must take into account the specific order modulation format in order to keep the computational complexity of the heuristic algorithms as low as possible. In our 16-QAM 1-opt LS MuD implementation, only candidate-vectors with minimum Euclidean distance from current solution, in a neighborhood Gray-coded scheme, are considered as described below.

Algorithm 2 M-QAM 1-opt LS-MUD

Input: $\mathbf{q}^{\text{MFB}}$;   **Output:** $\mathbf{q}^*$
begin
1. Initialize local search: $t = 1$,
     $\mathbf{q}^{\text{best}}[1] = \mathbf{q}^{\text{MFB}}$, the best initial solution;
2. for $t = 1, 2, \ldots$
     a. generate all feasible unitary Hamming distance neighbor of
        $\mathbf{q}^{\text{best}}[t]$, named $\mathbf{q}_i[t] \in \mathbf{Q}[t]$, $i = 1, \ldots mKI$;
     b. calculate $\Omega(\mathbf{q}_i[t])$, $\forall \mathbf{q}_i[t] \in \mathbf{Q}[t]$ using (7);
     c. if $\exists$ $\mathbf{q}_i[t]$ such that
        $[\Omega(\mathbf{q}_i[t]) > \Omega(\mathbf{q}^{\text{best}}[t])] \wedge [\Omega(\mathbf{q}_i[t]) > \Omega(\mathbf{q}_j[t]), j \neq i]$,
        than $\mathbf{q}^{\text{best}}[t+1] \leftarrow \mathbf{q}_i[t]$;
        else go to step 3;
   end
3. $\mathbf{q}^* = \mathbf{q}^{\text{best}}[t]$.
end

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$\mathbf{q}^{\text{MFB}}$: Rake output, replaced in real form.

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1) Minimal Distance Neighborhood Cases in M-QAM Constellation: Consider internal and peripherical symbols in a $16-$QAM constellation with $r_i = \{r_i^I, r_i^Q\} \in \mathcal{C} = \{\pm 1; \pm 3\}$. Figure 4 describes the neighborhood with minimum Euclidean distance. There are three neighborhood cases considered for the next generation in the M-QAM 1-opt LS-MUD algorithm:

a) four neighbors in an internal symbol-constellation case, when $r_i = \{\pm(< \mathcal{A}_{\max}), \pm(< \mathcal{A}_{\max})\}$, where $\mathcal{A}_{\max} = \max\{\mathcal{A}_{\text{real}}, \mathcal{A}_{\text{imag}}\}$, $r_i^I = \Re\{r_i\}$, and $r_i^Q = \Im\{r_i\}$:

$$\text{E.g.:} \quad r_i = \{1, 1\} \rightarrow \begin{cases} r_{i,n_{1,2}} = \left\{r_i^I, r_i^Q \pm 2\right\} \\ r_{i,n_{3,4}} = \left\{r_i^I \pm 2, r_i^Q\right\} \end{cases}$$

b.1) three neighbors in a peripheral symbol-constellation case, when $r_i = \{\pm \mathcal{A}_{\max}, \pm(< \mathcal{A}_{\max})\}$ or $\{\pm(< \mathcal{A}_{\max}), \pm \mathcal{A}_{\max}\}$:

$$\text{E.g.:} \quad r_i = \{3, 1\} \rightarrow \begin{cases} r_{i,n_{1,2}} = \left\{r_i^I, r_i^Q \pm 2\right) \\ r_{i,n_3} = \left\{\text{sgn}(r_i^I)(|r_i^I| - 2), r_i^Q\right\} \end{cases}$$

$$\text{or} \quad \rightarrow \begin{cases} r_{i,n_{1,2}} = \left\{r_i^I \pm 2, r_i^Q\right) \\ r_{i,n_3} = \left\{r_i^I, \text{sgn}(r_i^Q)(|r_i^Q| - 2)\right\} \end{cases}$$

b.2) two neighbors in a corner symbol-constellation case, when $r_i = \{\pm \mathcal{A}_{\max}, \pm \mathcal{A}_{\max}\}$:

$$\text{E.g.:} \quad r_i = \{3, 3\} \rightarrow \begin{cases} r_{i,n_1} = \left\{r_i^I, \text{sgn}(r_i^Q)(|r_i^Q| - 2)\right\} \\ r_{i,n_2} = \left\{\text{sgn}(r_i^I)(|r_i^I| - 2), r_i^Q\right\} \end{cases}$$
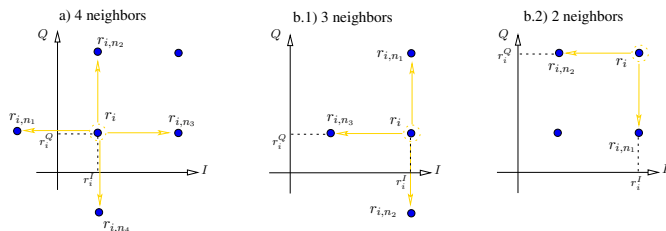

Fig. 4. The three cases of minimum distance neighborhood for $16-$QAM with regard to the current solution symbol, r_i.

V. NUMERICAL RESULTS

Next, the convergence and symbol-error-rate performance is analyzed by simulation, taking into account perfect and imperfect channel estimation at the receiver side. The HEUR-MUD SER performances are compared with the single-user bound (SuB), according to (15), since the OMUD computational effort results prohibitive for all system operations analyzed in this section.

A. SER Performance and Convergence Analysis

Monte-Carlo simulations (MCS) were carried out in order to verify the performance of the HEUR-MUDs in a 16-QAM asynchronous DS/CDMA system under multipath fading channel. MCS setup is summarized in the Appendix. The adopted system and channel parameters are described in the sequel.

- pseudo-noise (PN) spreading code,
- processing gain $N = 16$,
- one sampling per each chip period,
- carrier frequency $f_c = 2$GHz,
- maximum Doppler frequency $f_d = 222.2$ Hz,
- symbol rate $R = 384$ksymbols/s,
- rectangular pulse shaping $p(t)$ with unitary amplitude,
- decreasing power-delay profile with $L = 2$ paths [22], with $D = L$, and
- perfect coefficient estimates in the receiver, except for Section V-B.

Figure 5 shows the convergence curve for the HEUR-MUDs with additional system parameters: $K = 8$, $I = 3$ and SNR = 22dB. It is worth noting that the number of cost function evaluations per iteration is different for PSO-MUD and 1-opt LS-MUD, being 60 and 96, respectively[2]. Besides the different complexity, as discussed in section V-C, the PSO-MUD and 1-opt LS-MUD achieve different performances. It is possible to infer that 1-opt LS-MUD had a deficiency in escaping of local optimum, while PSO-MUD was able to achieve a near-optimum performance.
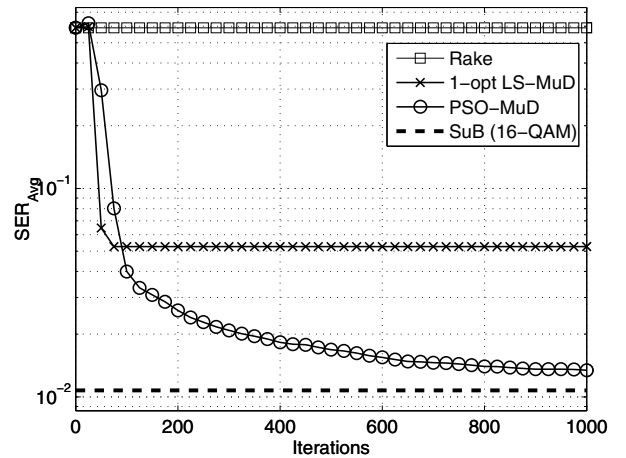

Fig. 5. Convergence performance of PSO- and 1-opt LS-MUD for medium system loading and moderate signal-to-noise ratio.

[2]For the 2-opt LS, this number becomes 9120.

Figure 6 shows the performance degradation of all HEUR-MUDs and Rake receiver as a function of increasing loading system $\mathcal{L} = \frac{K}{N}$. It is evident that both HEUR-MUDs have a superior performance than Rake receiver, for all evaluated loading. As can be seen, independently of the loading, PSO-MUD always accomplishes a better performance, although both 16-QAM HEUR-MUD schemes suffer a relative performance degradation for $\mathcal{L} > 0.5$.
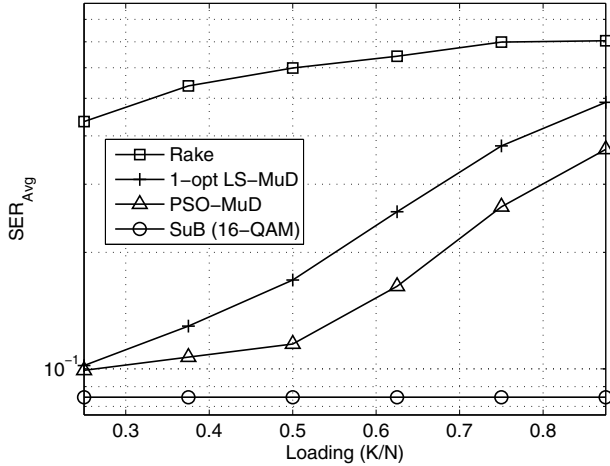

Fig. 6. HEUR-MUD performance in terms of system loading; SNR = 16dB, $D = 2$, $N = 16$.

B. Performance under Errors Estimates

Errors in the channels estimates were modeled through the continuous uniform distributions $\mathcal{U}[1 \pm \epsilon]$ centralized on the true values of the coefficients, resulting:

$$\widehat{\gamma}_{k,\ell}^{(i)} = \mathcal{U}[1 \pm \epsilon_\gamma] \times \gamma_{k,\ell}^{(i)}; \quad \widehat{\phi}_{k,\ell}^{(i)} = \mathcal{U}[1 \pm \epsilon_\phi] \times \phi_{k,\ell}^{(i)}, \quad (14)$$

where ϵ_γ and ϵ_ϕ are the maximum module and phase normalized errors for the channel coefficients, respectively.

For a low-moderate SNR and medium loading system ($\mathcal{L} = 0.5$), Figure 7 shows the performance degradation of the two HEUR-MUDs considering channel estimation errors of order of 10% or 25%, i.e., $\epsilon_\gamma = \epsilon_\phi = 0.10$ or $\epsilon_\gamma = \epsilon_\phi = 0.25$. One can see that, for medium system loading, an acceptable performance degradation is obtained if the error estimates are constrained by $\epsilon_{\gamma,\phi} < 0.1$.

Although there is a general performance degradation when error in channel coefficient estimates increasing, PSO-MUD remains with a better performance than 1-opt LS-MUD and Rake receivers under any error estimation case.

C. Complexity Analysis

As the cost function calculation (CFC) is the most significant factor, the number of CFC needed for the HEUR-MUD algorithms to converge is examined in more details. Considering the operation system of Figure 7 with $\epsilon_\gamma = \epsilon_\phi = 0$ condition, Table IV shows the necessary number of cost function calculations for the 16-QAM HEUR-MUDs convergence under different SNR conditions. The CFC number for OMUD is also
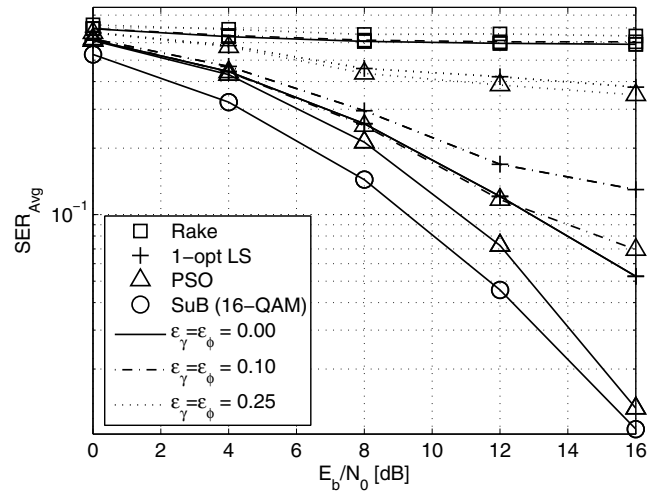

Fig. 7. Performance of HEUR-MUD considering errors in the channel coefficients estimates. $\mathcal{L} = 0.50$.

exposed for comparison purpose. As one can verify in Figure 5, 6, or 7, there is a performance gap between PSO-MUD and 1-opt LS-MUD, but with different computational complexities associated, according to Table IV. Therefore, in order to obtain a fair comparison between the two heuristic approaches, a different complexity comparison is suggested considering a partial PSO-MUD (pPSO-MUD): Figure 5 shows that until PSO-MUD reaches convergence, any additional iteration (with new $\mathcal{P}$ cost function calculations) implies in a performance gain, even if marginal (monotonic behavior); so, the pPSO-MUD evolves up to the iteration that occurs the crossing point with 1-opt LS-MUD curve (about 80th iteration, in that case).

TABLE IV
COMPUTATIONAL COMPLEXITY × SNR FOR 16-QAM HEUR-MUD UNDER $L = 2$ PATHS RAYLEIGH CHANNEL AND PERFECT CHANNEL ESTIMATES.

	Number of Cost Function Calculations (CFC)				
MUD	6 dB	10 dB	14 dB	18 dB	22 dB
1-opt LS	$2.4 \cdot 10^3$	$3.1 \cdot 10^3$	$2.9 \cdot 10^3$	$3.4 \cdot 10^3$	$3.6 \cdot 10^3$
PSO	$3.8 \cdot 10^3$	$1.2 \cdot 10^4$	$2.4 \cdot 10^4$	$3.6 \cdot 10^4$	$4.8 \cdot 10^4$
pPSO	$3.8 \cdot 10^3$	$4.9 \cdot 10^3$	$3.5 \cdot 10^3$	$3.6 \cdot 10^3$	$3.7 \cdot 10^3$
OMUD	$7.9 \cdot 10^{28}$				

The complexity advantage of 1-opt LS-MUD over PSO-MUD, which increases with SNR, can be explained since the gap between Rake and SuB become higher with SNR (Figure 7), the more sophisticated search procedure of the PSO-MUD is almost capable to overcome this gap for all range of evaluated SNR, while the simplest search approach of 1-opt LS-MUD reaches a worse bound, due to a lack of diversification and intensification mechanisms, and gets stuck on it. In summary, the 1-opt LS-MUD performance is much more susceptible to the start point choice.

Another important analysis is the increase complexity as function of the system loading. Figure 8 shows the behavior of both HEUR-MUDs as well as OMUD CFC-complexities. Comparing to PSO-MUD, again, 1-opt LS-MUD presents a smaller

complexity, but its performance is quite inferior. Considering the same performance, as explained in Table IV, the partial PSO complexity (labeled "pPSO-MuD" in Figure 8) becomes similar to 1-opt LS-MuD. Hence, different performance $\times$ complexity trade-off solutions (namely, partial and hybrid solutions) could be obtained, between pure 1-opt LS-MuD and PSO-MuD, depending on which is the most important factor to be considered in the system implementation.
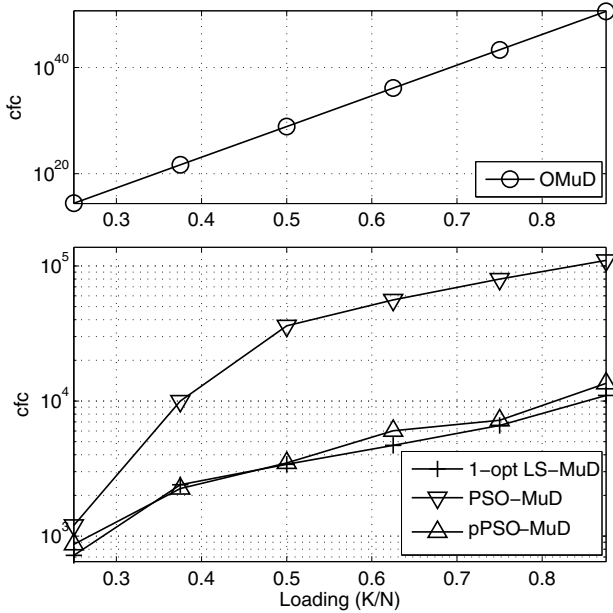

Fig. 8. Number of cost function calculations (CFC) for different HEUR-MuD and OMuD; $N = 16$.

Some idea about the feasibility of SISO HEUR-MuD implementation in hardware can be found in [26] for evolutionary algorithms. In [10], it was shown that PSO and evolutionary algorithms have equivalent complexity for SISO MuD under BPSK flat fading channels applications, mainly associated to cost function calculation operations.

VI. CONCLUSIONS

Under multipath channels and high-order (16-QAM) modulation, the PSO algorithm shows to be efficient for SISO MuD asynchronous DS/CDMA problem. The 1-opt LS algorithm, which is sufficient to deal with the MuD DS/CDMA problem under low-order modulation, showed to have a relative poor performance (lack of search diversity) under 16-QAM (or higher M) modulation, especially when the system loading and/or SNR increases.

Errors in channel estimates deteriorate similarly HEUR-MuDs performances, but they still remain superior with regard to Rake receiver, even with errors about 25%. In all evaluated system conditions, PSO-MuD resulted in the best performance solution. For a system under medium loading, a small performance degradation is observed when $\epsilon_{\gamma,\phi} < 0.05$.

Both HEUR-MuDs reach similar computational complexities, being much smaller than OMUD, for all systems conditions. The PSO-MuD is able to reach a better performance with an slightly increment in the computational complexity

when compared to the 1-opt LS-MuD. However, depending on the hardware resources available at the receiver side (feasible at base-station), and the system requirements (minimum throughput and maximum admitted symbol-error-rate), that additional complexity could be manageable.

The PSO-MuD accomplishes a flexible performance $\times$ complexity trade-off solutions (in the form of partial PSO-MuD), showing to be more appropriate that 1-opt LS for high-order modulation MuD asynchronous DS/CDMA systems under multipath channels conditions.

APPENDIX

A simplified diagram of the adopted Monte Carlo simulation setup is shown in Figure 9. In the random data generator block, the transmitted data were randomly generated and all M-QAM symbols had equal probability of being selected. The transmitter block implements Eq. (1), with the set of input variable $(A, I, K, N, M$ and spreading sequence type) adjusted according to the choices stated in Section V.
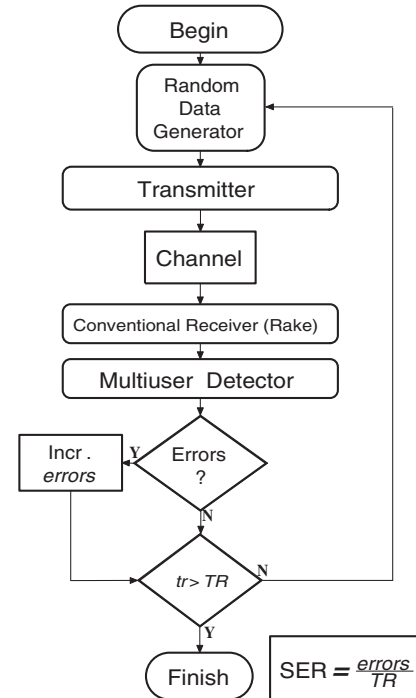

Fig. 9. Monte Carlo simulation diagram.

The channel block adds other stochastic characteristic to the model. Here the complex additive white Gaussian noise, with bilateral power spectral density equal to $N_0/2$, corrupts the received signal of all users. The power-delay profile for the Rayleigh fading channel can be adjusted by each user, admitting random delay and normalized power: $\sum_{\ell=1}^{L} \mathbb{E}[\gamma_\ell^2] = 1$. Then, the average SNR at the receiver input is given by: $\bar{E} = \sum_{\ell=1}^{L} \bar{E}_\ell$, where $\bar{E}_\ell = \text{SNR} \cdot \mathbb{E}[\gamma_\ell^2]$. For the exponential profile with 2 paths adopted in Section V, $\mathbb{E}[\gamma_1^2] = 0.8320$ and $\mathbb{E}[\gamma_2^2] = 0.1680$ were assumed, with the respective delays uniformly distributed on the interval $\tau_{k,\ell} \in [0; N-1]$.

The conventional receiver, with single-user detection capability (Rake receiver), performs combination of correlators output, eq. (5), and the estimated symbols are used as start point for the HEUR-MUD.

The minimal number of trials (TR) evaluated in the each simulated point (SNR) was obtained based on the single-user bound (SuB) performance. Considering a confidence interval, and admitting that a non-spreading and a spreading systems have the same equivalent bandwidth ($BW \approx \frac{1}{T_s} = BW_{\text{spread}} \approx \frac{N}{T_c}$), and thus, equivalently, both systems have the same channel response (delay spread, diversity order and so on), the SuB performance in both systems will be equivalent. So, the average symbol error rate for a single-user under M-QAM DS/CDMA system and L Rayleigh fading path channels with exponential power-delay profile and maximum ratio combining reception is found in [27, eq. (9.26)] as:

$$\text{SER}_{\text{SuB}} = 2\alpha \sum_{\ell=1}^{L} p_\ell (1 - \beta_\ell) + \qquad (15)$$

$$\alpha^2 \left[\frac{4}{\pi} \sum_{\ell=1}^{L} p_\ell \beta_\ell \times \tan^{-1}\left(\frac{1}{\beta_\ell}\right) - \sum_{\ell=1}^{L} p_\ell \right]$$

where:

$$p_\ell = \left(\prod_{k=1, k\neq\ell}^{L} \left(1 - \frac{\overline{\nu}_k}{\overline{\nu}_\ell}\right) \right)^{-1}, \quad \alpha = \left(1 - \frac{1}{\sqrt{M}}\right),$$

$$\beta_\ell = \sqrt{\frac{\overline{\nu}_\ell g_{\text{QAM}}}{1 + \overline{\nu}_\ell g_{\text{QAM}}}}, \quad g_{\text{QAM}} = \frac{3}{2(M-1)}, \quad \text{and}$$

$\overline{\nu}_\ell = \overline{\nu}_\ell^* \log_2 M = m\overline{\nu}_\ell^*$ denotes the average received signal-noise ratio per symbol for the ℓth path, with $\overline{\nu}_\ell^*$ being the correspondent SNR per bit per path.

Once the lower bound is defined, the minimal number of trials can be defined as:

$$\text{TR} = \frac{n_{\text{errors}}}{\text{SER}_{\text{SuB}}},$$

where the higher n_{errors} value, the more reliable will be the estimate of the SER obtained in MCS [28]. In this work, the minimum adopted $n_{\text{errors}} = 100$, and considering a reliable interval of 95%, it is assured that the estimate $\widehat{\text{SER}} \subset [0.823; 1.215]\,\text{SER}$. Simulations were carried out using MATLAB v.7.3 plataform, The MathWorks, Inc.

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