Resource Allocation and Packet Scheduling in OFDMA-Based Cellular Networks

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Abstract—**In this paper a study on different Radio Resource Management (RRM) methods employed in a multi-carrier cellular system is presented. This work focuses on the rate-adaptive resource allocation (sub-carriers and power), as well as the utility-based packet scheduling algorithms. The objective of the paper is to study the trade-off between system spectral efficiency and fairness among the users when the considered algorithms are used.**

I. Introduction

The wireless shared channel in cellular networks is a medium over which many Mobile Terminals (MTs) compete for resources. In such a scenario, spectral efficiency and fairness are crucial aspects for resource allocation. From a cellular operator perspective, it is very important to use the channel efficiently because the available frequency spectrum is scarce and the revenue must be maximized. From the users' point of view, it is more important to have a fair resource allocation so that they can meet their Quality of Service (QoS) requirements and maximize their satisfaction. The time-varying nature of the wireless environment, coupled with different channel conditions for different MTs, poses significant challenges to accomplishing these goals. In general, these objectives cannot be achieved simultaneously and an efficient trade-off must be achieved. In recent years Radio Resource Management (RRM) has been envisaged as one of the most efficient techniques to achieve a desirable trade-off among these two conflicting objectives in cellular multi-carrier systems.

Many next generation wireless systems are based on Orthogonal Frequency Division Multiple Access (OFDMA), which provides a high degree of flexibility that can be exploited by RRM algorithms. There are different sources of diversity in an OFDMA-based system, such as time, frequency and multi-user diversities. Following the path opened by the seminal article by Wong et al. [1], many Radio Resource Allocation (RRA) algorithms have been proposed to take advantage of these kinds of diversity, such as the dynamic allocation of subsets of sub-carriers for different MTs (Dynamic Sub-carrier Assignment (DSA)), and the adaptation of the Modulation and Coding Scheme (MCS) and power for each sub-carrier according to the instantaneous channel conditions (bit and power loading). Furthermore, Packet Scheduling (PSC) algorithms are responsible for deciding when the MTs will access the shared channel and with which transport format depending on the Channel State Information (CSI).

Many separate or joint RRA solutions including DSA, bit loading and power loading were based on combinatorial optimization. Most of the works in literature follow either the *margin adaptive* approach, formulating dynamic resource allocation with the goal of minimizing the transmitted power with a rate constraint for each user [2], or the *rate adaptive* approach aiming at maximizing the overall rate with a power constraint [3], [4]. In this latter case, the optimal solution for resource allocation in the downlink is often found as an application of the well-known waterfilling algorithm.

On the other hand, many works have been using Utility Theory to propose solutions for all the aforementioned RRA algorithms, including also multi-carrier PSC. The issues of efficiency, fairness and satisfaction of resource allocation have been well studied in economics, where utility functions are used to quantify the level of customers' satisfaction when the system allocates certain resources to them. Utility theory performs the optimization of a utility-pricing system, which is established based on the mapping of some performance criteria (e.g. rate, delay) or resource usage (e.g. sub-carriers, power) into the corresponding pricing values [5].

In this work, we will focus on the provision of Non-Real Time (NRT) services, such as World Wide Web (WWW) browsing, File Transfer Protocol (FTP) and e-mail. For these kind of services, the data rate is the most important QoS metric. The optimization problem can be formulated based on instantaneous or average data rates. The former case is stricter because QoS and fairness has to be guaranteed in each Transmission Time Interval (TTI), while the time window considered in the optimization problem based on average data rates adds a time diversity that relax the requirements on QoS and fairness.

The present work will be divided in two parts. In the first part, we will study rate adaptive sub-carrier and power allocation using optimization based on instantaneous data

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rates. In the second part, we will study multi-carrier packet scheduling using utility functions based on average data rates. The objective of the paper is to study the trade-off between system spectral efficiency and fairness among the users when the RRM algorithms mentioned above are used.

The paper is organized as follows. In section II the system model is described. Sections III-A and III-B present the mathematical formulation of the rate adaptive resource allocation based on instantaneous data rate and the packet scheduling based on utility theory and average data rates, respectively. The simulation results are depicted in section IV, while the conclusions are drawn in section V.

II. System Model

The considered scenario is a single cell with hexagonal shape. We consider a network with one transmitter (base-station) and J receivers (users). The transmitted Orthogonal Frequency Division Multiplexing (OFDM) signal is time-slotted, where in every time slot at most one user can be served over each sub-carrier. The considered environment is Typical Urban (TU) [6] where each user experiences independent transmit conditions. The channel is a frequency-selective Rayleigh fading channel, with the coherence time such that each sub-carrier experiences only flat fading. It is assumed that the channel fading rate is slow enough so that the frequency response does not change during a TTI interval. Each user also experiences shadowing with log-normal distribution. A perfect knowledge of the CSI at the transmitter side is assumed, with no signalling overhead transmitted. The signal strength at the receiver side depends on the path-loss calculated by:

$$L = 128.1 + 37.6 \log_{10} d, \tag{1}$$

where d is the distance to the base station in km.

The bit allocation on each sub-carrier is determined using the Shannon's capacity model shown in (2) below [5]:

$$c_{j,k} = \log_2 \left(1 + \Gamma p_k \rho_{j,k}\right) \tag{2}$$

where $c_{j,k}$ is the achievable throughput of user j over sub-carrier k, p_k is the transmit power allocated at sub-carrier k, $\rho_{j,k}$ is the Signal-to-Noise Ratio (SNR) of user j at sub-carrier k, and Γ is the SNR gap given by $\frac{1.5}{-\ln 5BER}$ [5] (the considered target Bit Error Rate (BER) was 10^{-6}).

It was assumed that the MTs remained stationary, hence there is no need to implement any handover scheme. All users are assumed to have an infinite amount of data to transmit during the whole simulation run (full-buffer model).

III. Resource Allocation Algorithms

A. Rate adaptive sub-carrier and power allocation based on instantaneous data rates

RRA often leads to algorithms whose implementation is very complex. In fact the allocation problem is in general not convex since the allocation variable is integer and can assume only two values: 1 when the channel is allocated to a specific user and 0 otherwise. In most cases the optimal solution can be found only evaluating all possible allocations and the complexity grows exponentially in the number of users and sub-carriers. Therefore, most of the literature has been focused on the development of sub-optimal heuristics that have a lower computational complexity but that still yield good results. Many algorithms make the problem convex by relaxing the integer constraint on the allocation variable. Unfortunately, non-integer solutions are hardly applicable in many scenarios where a sub-carrier should be actually allocated or not to a user. In the following we will focus on the RRA problem outlining its most common formulations and solutions.

1) Sum Rate Maximization: The most common mathematical formulation of the RRA problem is

$$\begin{aligned} &\max_{\mathbf{p},\mathbf{x}} \sum_j \sum_k c_{j,k} \cdot x_{j,k} \\ &\qquad s.t. \\ &\sum_j x_{j,k} \le 1 \quad \forall k \\ &\sum_j \sum_k p_{j,k} \le P_{\max} \\ &x_{j,k} \in \{0,1\} \quad \forall j,k \end{aligned} \tag{3}$$

where $P_{\max}$ is the maximum allowed transmit power of the Base Station (BS). The optimization variables are $\mathbf{x}$, the vector of the allocations, and $\mathbf{p}$, the vector containing the power levels of all sub-carriers.

In its original formulation the problem (3) has been solved in [3] by assigning each sub-carrier to the user that maximizes its gain on it and then performing waterfilling over all the sub-carriers. On one hand, such a solution maximizes the cell throughput but on the other hand is extremely unfair tending to privilege the users that are closest to the BS and neglecting all the others.

2) Max-Min Rate Adaptive: The RRA allocation (3) tends to starve the users with the worse channel gains, i.e. the users that are more distant from the BS. Thus, in [4] the RRA problem has been formulated with the goal of maximizing the minimum capacity offered to each user, thus introducing fairness among the users. In general, fairness among the MTs comes at the cost of a decreased overall throughput of the cell. The max-min RRA problem is formulated as follows

$$\begin{aligned} &\max_{\mathbf{p},\mathbf{x}} \min_j c_{j,k} \cdot x_{j,k} \\ &\qquad s.t. \\ &\sum_j x_{j,k} \le 1, \quad \forall k \\ &\sum_j \sum_k p_{j,k} \le P_{\max} \\ &x_{j,k} \in \{0,1\} \quad \forall j,k \end{aligned} \tag{4}$$

Unfortunately, the problem in the formulation (4) is not convex and the authors in [4] study an heuristic that is based on: a) transmitting the same amount of power ($P_{\max}/K$) on each channel; b) implementing an *assignment* strategy that iteratively assigns each sub-carrier to the user with the smallest rate.

3) Sum Rate Maximization with Proportional Rate Constraints: The max-min RRA (4) guarantees that all users achieve a similar data rate. However, different users may require different data rates. In this case the max-min solution is not able to comply with the different user requirements. The RRA algorithm presented in [7] is designed to allocate radio resources proportionally to different rate constraints that reflect different levels of service. The RRA problem is formulated as follows

$$\begin{array}{c} \max_{\mathbf{p},\mathbf{x}} \sum_j \sum_k c_{j,k} \cdot x_{j,k} \\ s.t. \\ \sum_j x_{j,k} \leq 1, \quad \forall k \\ \sum_j \sum_k p_{j,k} \leq P_{\max} \\ x_{j,k} \in \{0,1\} \quad \forall j,k. \\ R_1 : R_2 : ... : R_J = \gamma_1 : \gamma_2 : ... : \gamma_J \end{array} \tag{5}$$

where R_j indicates the rate for user j, defined as $R_j = \sum_k c_{j,k} \cdot x_{j,k}$ and γ_j $(j = 1, ..., J)$ is a set of predetermined values that are used to ensure proportional fairness among users. The optimization in (5) is a mixed binary integer programming problem and as such is in general very hard to solve. Thus, also in this case the problem is solved using a suboptimal heuristic and the optimization (5) is performed in two steps. In the first step, following the approach taken in [4], the sub-carriers are allocated trying to comply as much as possible with the proportional rate constraints and assuming a uniform power distribution. In the second step, having fixed the sub-carrier allocation, the power is distributed to the users so that the proportional rate constraints are met exactly.

B. Packet Scheduling Based on Utility Theory and Average Data Rates

In this section we formulate PSC algorithms that use Utility Theory in order to find an efficient trade-off between system spectral efficiency and fairness among the users. The considered optimization problem is the maximization of the total utility with respect to the throughput (average data rate), which is calculated using a low-pass Simple Exponential Smoothing (SES) filtering as indicated in (6) [5].

$$T_j\left[n\right] = \left(1 - \frac{1}{t_f}\right) \cdot T_j\left[n-1\right] + \left(\frac{1}{t_f}\right) \cdot r_j \tag{6}$$

where r_j is the instantaneous data rate of the jth MT and t_f is a filtering time constant.

Assuming that the time constant of the exponential filter is sufficiently large, it is proven in [5] that the DSA problem has a closed form solution. The MT j^* is chosen to transmit on the kth sub-carrier in TTI n if it satisfies the condition given by (7):

$$j^* = \arg\max_j \left\{ U_j^{'}\left(T_j\left[n-1\right]\right) \cdot c_{j,k}\left[n\right] \right\}, \quad \forall j \tag{7}$$

where $U_j^{'}(.)$ is the marginal utility of the jth MT, $T_j\left[n-1\right]$ is the throughput of the jth MT up to TTI $n-1$, and $c_{j,k}\left[n\right]$ denotes the instantaneous achievable transmission efficiency of the jth MT on the kth sub-carrier.

We will consider a family of utility functions of the form presented in (8) below [8].

$$U_j\left(T_j\left[n\right]\right) = \frac{T_j\left[n\right]^{1-\alpha}}{1-\alpha} \tag{8}$$

where α is a non-negative parameter that determines the degree of fairness. The fairness of the utility function becomes stricter as α increases.

According to (7), this is equivalent to consider a priority function of the PSC algorithm given by:

$$P_{j,k}^{PSC} = \frac{c_{j,k}\left[n\right]}{T_j\left[n-1\right]^{\alpha}}, \quad \forall j,k; \quad \alpha \in [0,\infty) \tag{9}$$

For each of the K sub-carriers in the system, a multi-carrier PSC algorithm calculates the priority functions for all J MTs according to (9) and assign it to the MT that has the highest priority value.

We will show in sections III-B1, III-B2 and III-B3 that, depending on the value of the parameter α, the general utility framework presented above can be designed to work as any of three well-known classical PSC algorithms: Max-Rate (MR), Max-Min Fairness (MMF) and Proportional Fairness (PF). Furthermore, in section III-B4 we present the Adaptive Fairness (AF) PSC algorithm, which can achieve an adaptive trade-off between spectral efficiency and fairness according to the cellular operator's objectives.

1) Rate Maximization: The MR PSC algorithm is able to maximize the system spectral efficiency because it considers a linear utility function $U_j\left(T_j\left[n\right]\right) = T_j\left[n\right]$, which yields a constant marginal utility $U_j^{'}\left(T_j\left[n\right]\right) = 1$ [5]. One can notice that this can be achieved setting $\alpha = 0$ in (8). According to (9), this is equivalent to consider a priority function related to the MR algorithm given by (10) below.

$$P_{j,k}^{MR} = c_{j,k}\left[n\right], \quad \forall j,k \tag{10}$$

As the final result, each sub-carrier will be assigned to the MT that has the highest channel gain on it. The MR criterion maximizes the system capacity at the cost of unfairness among the MTs, because those with poor radio link quality will probably not have chance to transmit.

2) Max-Min Fairness: The utility function of the MMF algorithm is the limit of the function in (8), when $\alpha \to \infty$ [8]. According to (7) and (9), the priority function is dependent on the marginal utility $U_j^{'}\left(T_j\left[n\right]\right)$ and the achievable instantaneous transmission efficiency $c_{j,k}\left[n\right]$. However, in the case of the MMF criteria and when considering MTs with lower data rates, the influence of the marginal utility when $\alpha \to \infty$ is so high that the influence of the channel quality becomes negligible. Taking this fact into account, we can assume a more simplified priority function for the MMF algorithm given in (11), which is also known in the literature as the "Fair Throughput" criterion [9].

$$P_{j,k}^{MMF} = \frac{1}{T_j\left[n-1\right]}, \quad \forall j,k \tag{11}$$

which gives priority to the MT that has experienced the worst throughput so far. In this way, in terms of throughput, it is the most fair criterion possible, since all MTs will have approximately the same throughput in the long-term. However, since this criterion maximizes the throughput of the worst MTs, it will provide low aggregate system throughput.

3) Proportional Fairness: A trade-off between spectral efficiency and fairness can be achieved by means of a PF PSC algorithm [10]. In utility theory, the logarithmic utility function is associated with the proportional fairness [5]. In the general family of utility functions presented in (8), the logarithmic function can be achieved when $\alpha \rightarrow 1$ (see proof on [8]). Therefore, according to (9), the priority function of the PF algorithm is given by (12).

$$P_{j,k}^{PF} = \frac{c_{j,k}\left[n\right]}{T_j\left[n-1\right]}, \quad \forall j, k \tag{12}$$

4) Adaptive Fairness: The AF PSC algorithm, which was proposed in [11], joins in a unified framework the three aforementioned classical PSC algorithms (MR, MMF and PF). In the light of utility theory, it was shown that a general PSC algorithm based on (8) is able to provide several degrees of fairness. The AF algorithm adaptively explores this flexibility in order to achieve an efficient trade-off between spectral efficiency and fairness planned by the network operator. However, it is difficult to design an adaptive control of the α parameter because it is defined over a large range of values. Instead of that, the AF algorithm transforms the priority function of (9) into another priority function that is based on a parameter β, which is defined over a controlled range and provides the possibility of a stable and simple adaptive control. The priority function of the the AF algorithm is presented in (13) below.

$$P_{j,k}^{AF} = \frac{c_{j,k}\left[n\right]^{1-\beta}}{T_j\left[n-1\right]^{\beta}}, \quad \forall j, k; \quad \beta \in [0, 1] \tag{13}$$

Notice that in a conceptual point of view, the priority functions on (9) and (13) perform in the same way. The AF algorithm is able to work as the classical PSC algorithms by means of the adaptation of the β parameter. The values of $\beta = \{0, 0.5, 1\}$ corresponds to the MR, PF and MMF, respectively.

The AF algorithm is based on the definition of a fairness index ϕ_j, which is based on throughput and calculated for each MT in the cell. The user fairness index changes with time and is defined as:

$$\phi_j = \frac{T_j\left[n-1\right]}{T_j^{req}} \tag{14}$$

where T_j^{req} is the throughput requirement of the jth MT. Next, we define a fairness index for the whole system, which is given by (15) [12].

$$\Phi = \frac{\left(\sum_{j=1}^{J} \phi_j\right)^2}{J \cdot \sum_{j=1}^{J} \left(\phi_j\right)^2} \tag{15}$$

where J is the number of MTs in the cell and ϕ_j is the fairness index of the jth MT given by (14). Notice that $0 \leq \Phi \leq 1$. A perfect fair allocation is achieved when $\Phi = 1$, which means that the throughput allocated to all MTs are equally proportional to their throughput requirements (all user fairness indexes are equal). The worst allocation occurs when $\Phi = 1/J$, which means that all sub-carriers were allocated to only one MT.

The objective of the AF algorithm is to assure a strict fairness distribution among the MTs, i.e. the system fairness index Φ must be kept around a planned value Φ_{target}. Therefore, the AF algorithm adapts the parameter β in the scheduling policy presented in (13) in order to achieve the desired operation point. In order to do that, the new value of the parameter β is calculated using a feedback control loop of the form:

$$\beta\left[n\right] = \beta\left[n-1\right] - \eta \cdot \left(\Phi_{filt}\left[n\right] - \Phi_{target}\right) \tag{16}$$

where $\Phi_{filt}\left[n\right]$ is a filtered version of the system fairness index using a SES filtering, Φ_{target} is the desired value for the index, and the parameter η is a step size that controls the adaptation speed of the parameter β. A SES filter, which is suitable for time series with slowly varying trends, was used to suppresses short-run fluctuations and smooth the time series $\Phi\left[n\right]$.

IV. Simulation Results

In this section the simulation-specific parameters as well as the simulation results are presented. The simulation results for the resource allocation and packet scheduling algorithms are depicted in sections IV-A and IV-B, respectively. The main simulation parameters are presented in Table I.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Number of cells	1
Maximum BS transmission power	1 W
Cell radius	500 m
MT speed	static
Carrier frequency	2 GHz
Number of sub-carriers	192
Sub-carrier bandwidth	15 kHz
Path loss	using (1)
Log-normal shadowing standard dev.	8 dB
Small-scale fading	Typical Urban (TU)
AWGN power per sub-carrier	-123.24 dBm
BER requirement	10^{-6}
Link adaptation	continuous using (2)
Transmission Time Interval (TTI)	0.5 ms
Traffic model	Full buffer
Throughput filtering time constant (t_f)	50
Minimum β value	0
Maximum β value	1
AF PSC control time window	0.5 ms
AF PSC target fairness index (Φ_{target})	0.5 or 0.9
AF PSC step size (η)	0.1
AF PSC filtering time constant	10

The metrics used for evaluation and comparison of the investigated resource allocation algorithms were:

- Total cell throughput (resource allocation efficiency factor);
- Fairness index, according to (15).

A. Rate Adaptive Sub-carrier and Power Allocation

The results presented in this section are obtained for all RRA algorithms presented in Section III-A averaged over 500 realizations. For the sum rate maximization with proportional rate constraints two different sets of rate constraints have been studied: the case where the rate constraints are set equal for all users (Prop rate 1), i.e. $\gamma_j = 1$ ($j = 1, ..., J$), and the case where the rate constraints are set proportional to the user pathloss (Prop rate 2).

Fig. 1 shows the total cell throughput for the different algorithms: the sum rate maximization algorithm achieves the highest throughput and the max-min the lowest. The results show also the flexibility of the algorithm with proportional rate constraints. As expected, when the set of rate constraints are all equal its behavior is almost identical to the max-min algorithm. On the other hand, when the system tends to favor the users nearer to the BS, the throughput approaches the sum rate results.

Fig. 2 shows the mean fairness index (according to the definition in (15)) for the various RRA algorithms. In this case the max-min and the algorithm with equal rate constraints outperform all the others. The algorithm with rate constraints proportional to the pathloss, even if it guarantees access to all users, is not very fair. This is due to the fact that in our simulation setting, the difference in pathloss can be several orders of magnitude large. Thus, users close to cell boundaries will have a much smaller throughput than users near the BS.

B. Utility-Based Packet Scheduling

In case of the utility-based packet scheduling analysis, the power distribution over all sub-carriers was uniform with no power adaptation. For each simulation point 10 different realizations have been considered, with the simulation time span for each of the realizations set to 30 s (60000 TTIs).

Fig. 3 shows the system fairness index calculated using (15) for different cell loads. We run simulations with two different AF target fairness indexes: 0.5 and 0.9. It can be observed that AF is successful to achieve its main objective, which is to guarantee a strict fairness distribution among the MTs. This is achieved due to the feedback control loop that dynamically adapts the parameter β of the AF priotity function (see (13)).

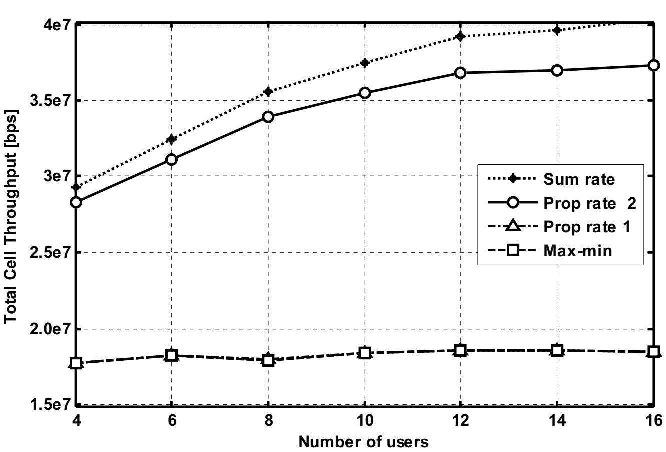

Fig. 1. Achievable throughput as function of the number of users

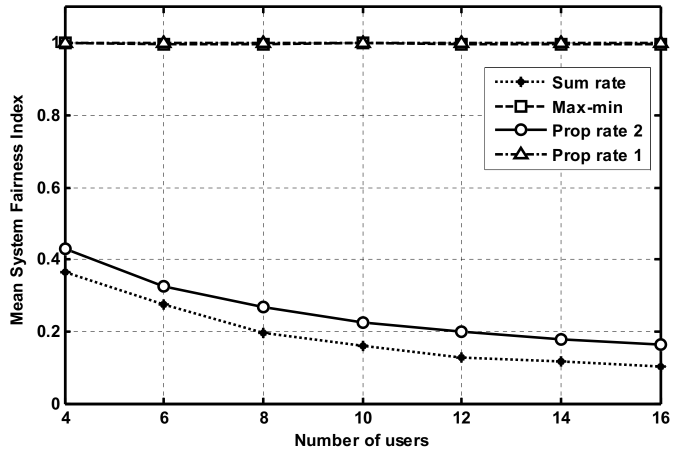

Fig. 2. Measured fairness as function of the number of users

As expected, MMF provided the highest fairness, very close to the maximum value of 1, while MR proved to be the most unfair strategy. PF presented a trade-off between MMF and MR. The advantage of the AF algorithm in comparison with the others is that it can be designed to provide any required fairness distribution, while the classical PSC strategies are static and do not have the freedom to adapt themselves and guarantee a specific performance result.

The total cell throughput for different cell loads is shown in Fig. 4. As expected, MR was able to maximize the spectral efficiency, while MMF presented the lowest cell throughput. Since PF is a trade-off between MR and MMF, its performance lied between them. Looking at Fig. 3, one can expect that depending on the value of the AF target fairness index, the AF resource efficiency would be somewhere in the middle between the performances of MMF, PF and MR. This can be observed in Fig. 4. On one hand, when the AF target fairness index is set to 0.5, AF works as an hybrid scheduling policy between PF and MR. On the other hand, the AF performance

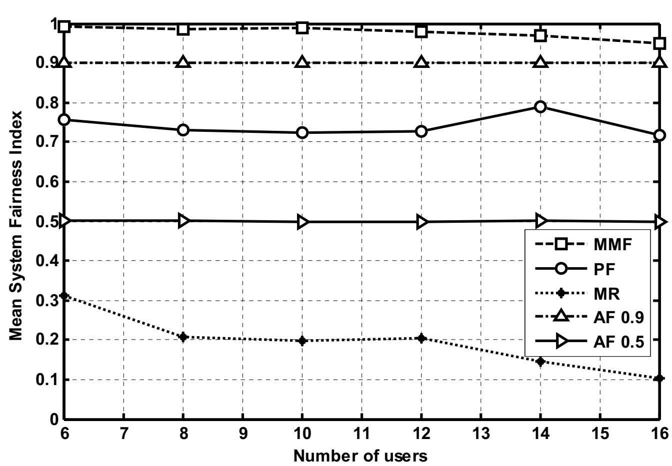

Fig. 3. Comparison of utility-based packet scheduling algorithms regarding the system fairness index

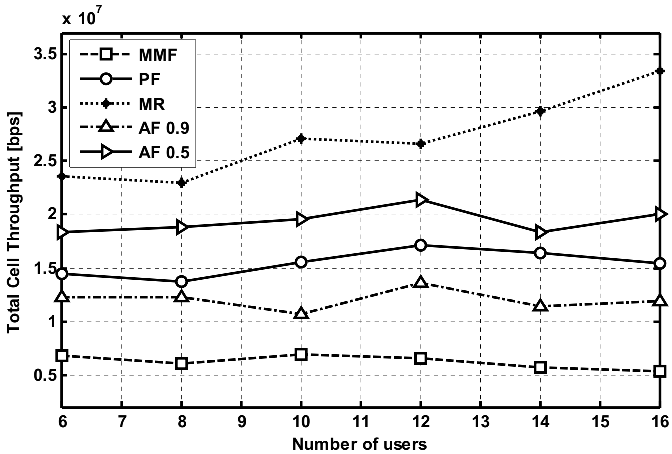

Fig. 4. Comparison of utility-based packet scheduling algorithms regarding the total cell throughput

in terms of total cell throughput lies between MMF and PF when the target fairness index is set to 0.9.

V. Conclusions

In this paper we investigated the trade-off between system spectral efficiency and fairness among users in OFDMA-based cellular networks. Two RRM approaches were studied: rate adaptive resource allocation (sub-carriers and power) based on instantaneous data rate and utility-based packet scheduling based on average data rate (throughput). Comparing the two approaches, one can see clearly the direct relationship between sum rate maximization RRA and MR PSC, and also max-min RRA and MMF PSC. Furthermore, possible trade-offs were presented, such as sum rate maximization with proportional rate constraints in the case of RRA, and PF and AF in the case of PSC.

It was concluded from simulation results in a single-cell scenario that is possible to achieve an efficient trade-off between system spectral efficiency and fairness using any of the two RRM approaches. The total cell throughput and fairness index presented by the rate adaptive RRA were higher than the utility-based PSC because the former used power adaptation.

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